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A PROPOSED REFORMULATION OF THE ELECTROMAGNETIC EQUATIONS AND REVISION OF UNITS.

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NOTATION.

Length	l
Area	A
Velocity	v
Velocity of light (ratio of e.m.u. to e.s.u. of charge)	c
Force	K
Force per unit volume	F
Energy	W
Power	$\mathcal{P}$
Electric charge density or charge per unit volume	ρ
Magnetic charge density or pole strength per unit volume	τ
Electric charge or quantity of electricity	q
Magnetic charge or pole strength	m
Electric intensity or field strength	E
Magnetic intensity or field strength	H
Electric displacement or electric flux density	D
Magnetic induction or magnetic flux density	B
Electric flux (of displacement)	ψ
Magnetic flux (of induction)	ϕ
Electric moment	$\mathfrak{p}$
Magnetic moment	$\mathfrak{M}$
Polarization (electric moment per unit volume)	P

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Magnetization (magnetic moment per unit volume)	I
Electromotive force (e.m.f.)	$\mathcal{E}$
Magnetomotive force (m.m.f.)	$\mathcal{F}$
Current	i
Current density (current per unit cross-section)	j
Resistance	R
Capacitance	C
Inductance	L
Reluctance	$\mathcal{R}$

OBJECTIVES.

Any attempt to improve the formulation of the electromagnetic equations and the selection of units to be used with them will be abortive if it aims at an ideal solution which ignores present usage and the present practical requirements of both the physicist and the engineer. A careful study of all possible re-formulations of the equations and revisions of the units in use today, made with the essential requirements of the theoretical physicist on the one hand and those of the engineer and experimenter on the other continually in view, leads to the conclusion that the adoption of a single set of equations and a single system of units by both groups is impracticable. Any such proposal would be unsatisfactory to one group or to the other or to both, and no exercise of authority could force it upon the two groups.

Hence practicable efforts for improvement of the present chaotic situation must recognize the need of two sets of equations and two systems of units, one devised to fulfill the requirements of the physicist and the other to meet those of the engineer. At the same time the two systems of units should be so chosen as to keep as simple as possible the transformations from the one to the other.

An essential criterion which should be fulfilled by both sets of equations is that they should be cast in such a form as to exhibit the symmetry existing between electric quantities on the one hand and magnetic quantities on the other. The advantages of symmetry have been discussed by one of us in a previous paper.[1] Briefly put they are:

(1) All derived equations are symmetrical in electric and

[1] Leigh Page, National Research Council—Bulletin 93.

magnetic quantities. Thus only half the usual number of different coefficients appear.

(2) Corresponding electric and magnetic quantities have the same physical dimensions.

(3) In transforming from the physicist's units to the engineer's units, the transformation factors are the same for corresponding electric and magnetic quantities. This halves the number otherwise present.

(4) The definitions of corresponding electric and magnetic quantities correspond arithmetically.

Of these advantages (3) is of especial importance once it is recognized that two systems of units constitute the minimum practicable number.

SYMMETRICAL EQUATIONS.

The most general symmetrical form in which the electromagnetic equations can be put [2] is

$$\text{Div } \mathbf{D} = 4\pi\frac{\alpha}{\gamma}\rho, \quad (1) \qquad \text{Div } \mathbf{B} = 0, \quad (2)$$

$$\text{Curl } \mathbf{E} = -\frac{1}{c}\alpha\gamma\dot{\mathbf{B}}, \quad (3) \qquad \text{Curl } \mathbf{H} = \frac{1}{c}\alpha\gamma\dot{\mathbf{D}} + \frac{4\pi\alpha^2}{c}\rho\mathbf{v}, \quad (4)$$

$$\mathbf{D} = \frac{1}{\alpha\gamma}\mathbf{E} + 4\pi\frac{\alpha}{\gamma}\mathbf{P} = \frac{\kappa}{\alpha\gamma}\mathbf{E}, \quad (5)$$

$$\mathbf{B} = \frac{1}{\alpha\gamma}\mathbf{H} + 4\pi\frac{\alpha}{\gamma}\mathbf{I} = \frac{\mu}{\alpha\gamma}\mathbf{H}, \quad (6)$$

$$\mathbf{F} = \rho\left\{\mathbf{E} + \frac{\alpha\gamma}{c}\mathbf{v} \times \mathbf{B}\right\}, \quad (7)$$

where α and γ are arbitrary coefficients which may be dimensional.

The Heaviside-Lorentz equations are obtained by making $\alpha = 1/(4\pi)^{1/2}$, $\gamma = (4\pi)^{1/2}$ and the Gaussian equations by making $\alpha = \gamma = 1$. Neither the equations in electromagnetic units nor in electrostatic units, as given by Maxwell, are symmetrical, and hence they cannot be obtained from the above formulation.

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[2] Leigh Page, Physics, **2**, p. 289, 1932.

THE PHYSICIST'S EQUATIONS AND UNITS.

Unquestionably the symmetrical Heaviside-Lorentz equations, used by the late H. A. Lorentz in his *Lehrbuch der Physik*, *Theory of Electrons* and other works, appeal most strongly to the theoretical physicist by virtue of their unparalleled simplicity. Nevertheless the use of Maxwell's electrostatic and electromagnetic units is so general that it seems more practicable to recommend the symmetrical Gaussian equations. When c.g.s. units are used with these equations all electric quantities (except displacement) are measured in electrostatic units and all magnetic quantities in electromagnetic units. The only inferiority of the Gaussian equations as compared with the Heaviside-Lorentz equations lies in the presence of a few 4π's. If we designate quantities measured in c.g.s. Gaussian units by the subscript g, the Gaussian equations are

$$\text{Div}_g \mathbf{D}_g = 4\pi\rho_g, \quad (8) \qquad \text{Div}_g \mathbf{B}_g = 0, \quad (9)$$

$$\text{Curl}_g \mathbf{E}_g = -\frac{\mathrm{I}}{c_g}\dot{\mathbf{B}}_g, \quad (10) \qquad \text{Curl}_g \mathbf{H}_g = \frac{\mathrm{I}}{c_g}(\dot{\mathbf{D}}_g + 4\pi\rho_g\mathbf{v}_g), \quad (11)$$

$$\mathbf{D}_g = \mathbf{E}_g + 4\pi\mathbf{P}_g = \kappa\mathbf{E}_g, \quad (12)$$

$$\mathbf{B}_g = \mathbf{H}_g + 4\pi\mathbf{I}_g = \mu\mathbf{H}_g, \quad (13)$$

$$\mathbf{F}_g = \rho_g\left\{\mathbf{E}_g + \frac{\mathrm{I}}{c_g}\mathbf{v}_g \times \mathbf{B}_g\right\}. \quad (14)$$

Here F_g is in dynes/cm.^{3}, v_g in cm./sec. and c_g is $3(10)^{10}$ cm./sec. (more accurately $2.99796(10)^{10}$ cm./sec.).

These equations would probably be much preferred to the Heaviside-Lorentz equations by the experimental physicist since they use the e.s.u. and e.m.u. of Maxwell with which he is most familiar. On the other hand they would not be seriously objected to by the theoretical physicist in spite of the fact that they are not quite so simple in form as the Heaviside-Lorentz equations.

THE ENGINEER'S EQUATIONS AND UNITS.

The engineer's prime requisites are that his equations (*a*) should utilize the eight internationally recognized practical units, (*b*) should be based on the meter, kilogram and second

as fundamental units. With these fundamental units the joule and the watt are the units of energy and power respectively, and in order to bring the coulomb, ampere, volt, ohm, farad and henry into the system it is necessary only to insure that the coulomb be the unit of charge, since the remaining units are defined in terms of the joule, watt, coulomb and second.

As it follows from (1), (5) and (7) that the law of force between point charges q and q' at rest is

$$K = \alpha^2 \frac{qq'}{\kappa r^2}, \tag{15}$$

the dimensions of charge are

$$[q] = \left[\frac{M^{1/2}L^{3/2}}{\alpha T}\right].$$

We shall designate quantities measured in practical m.k.s. units by the subscript p to distinguish them from quantities measured in Gaussian c.g.s. units, which are indicated by the subscript g. Then as $\alpha_g = 1$, it follows from (15) that

$$q_g = \alpha_p q_p, \tag{16}$$

where α_p may be a dimensional constant. If we designate the numerical measure of a quantity by a bar over the letter denoting the quantity, this equation may be written

$$\bar{q}_g \frac{\text{gm.}^{1/2}\ \text{cm.}^{3/2}}{\text{sec.}} = \bar{\alpha}_p \bar{q}_p \frac{\text{kg.}^{1/2}\ \text{meter}^{3/2}}{\text{sec.}}$$

$$= (10)^{9/2}\, \bar{\alpha}_p \bar{q}_p \frac{\text{gm.}^{1/2}\ \text{cm.}^{3/2}}{\text{sec.}},$$

whence

$$\bar{q}_g = (10)^{9/2}\, \bar{\alpha}_p \bar{q}_p.$$

Now since the Gaussian unit of charge is the e.s.u. it is necessary that $\bar{q}_g = 3(10)^9\, \bar{q}_p$ in order that the coulomb should be the practical m.k.s. unit of charge. More accurately, of course, this relation should be $\bar{q}_g = 2.99796(10)^9\, \bar{q}_p$, but for purposes of illustration it is more convenient to use the round number $3(10)^9$ corresponding to the approximate value $3(10)^{10}$ cm./sec. for the velocity of light and to ignore the slight in-

accuracy of less than one-tenth of one per cent. Consequently $\bar{\alpha}_p = 3(10)^{9/2}$.

This relation indicates that the inclusion of the eight internationally recognized practical units in a comprehensive m.k.s. system of units utilizing symmetrical equations determines the numerical value of α_p alone. Its physical dimensions as well as the value of the constant γ in the most general symmetrical formulation (1)–(7) of the electromagnetic equations are entirely arbitrary.

It seems desirable to make the coefficient $4\pi\alpha^2/c$ in Ampère's law (4) a pure number as in e.m.u. Then the dimensions of α_p must be $[L^{1/2}/T^{1/2}]$. The numerical part of α_p is fixed by our previous requirements and cannot be altered. Next it appears advantageous to make the coefficient of $\dot{\mathbf{B}}$ in the much used law (3) of current induction equal to unity as in Maxwell's formulation. This makes $\gamma_p = c_p/\alpha_p$. So we arrive at the solution

$$\left.\begin{aligned} \alpha_p &= 3(10)^{9/2}\ \text{meter}^{1/2}/\text{sec.}^{1/2} = \sqrt{30c_p}, \\ \gamma_p &= (10)^{7/2}\ \text{meter}^{1/2}/\text{sec.}^{1/2} = \sqrt{c_p/30}, \end{aligned}\right\} \tag{17}$$

since $c_p = 3(10)^8$ meter/sec. These considerations lead to the equations

$$\text{Div}_p\, \mathbf{D}_p = 120\pi\rho_p, \tag{18}$$

$$\text{Div}_p\, \mathbf{B}_p = 0, \tag{19}$$

$$\text{Curl}_p\, \mathbf{E}_p = -\dot{\mathbf{B}}_p, \tag{20}$$

$$\text{Curl}_p\, \mathbf{H}_p = \dot{\mathbf{D}}_p + 120\pi\rho_p\mathbf{v}_p, \tag{21}$$

$$\mathbf{D}_p = \frac{\text{I}}{c_p}\mathbf{E}_p + 120\pi\mathbf{P}_p = \frac{\kappa}{c_p}\mathbf{E}_p, \tag{22}$$

$$\mathbf{B}_p = \frac{\text{I}}{c_p}\mathbf{H}_p + 120\pi\mathbf{I}_p = \frac{\mu}{c_p}\mathbf{H}_p, \tag{23}$$

$$\mathbf{F}_p = \rho_p\{\mathbf{E}_p + \mathbf{v}_p \times \mathbf{B}_p\}. \tag{24}$$

Here F_p is measured in units of $(10)^5$ dyne/meter^3, v_p in meter/sec. and c_p is $3(10)^8$ meter/sec. (more accurately $2.99796(10)^8$ meter/sec.).

In these equations the coefficient 120π should be written more accurately $4\pi\,(29.9796) = 376.735$. In fact it might be advisable in printing the fundamental equations and derived

equations in texts, handbooks, etc. to write this coefficient in the convenient exact form $40\pi\epsilon$, where $\epsilon = 2.99796$.

It may be noted that the ampere-turn is *not* the unit of magnetic field strength; indeed the inclusion of the ampere-turn with the other practical units would thwart all efforts to attain the desired solution in terms of a set of symmetrical equations, for the numerical value of the constant $4\pi\alpha^2/c$ in Ampère's law (4) is fixed by the requirement that the eight internationally recognized practical units be included in the proposed m.k.s. system. Incidentally these equations leave the way open to fix arbitrarily *one* of the six internationally recognized practical electric units (such as the ohm) by changing the value of the numerical coefficient 376.735, if such action is at any time thought desirable. Finally it may be mentioned that the proposed equations remain unchanged if the cm., $(10)^7$ gm. and sec. are chosen as fundamental units instead of the meter, kg. and sec., as has been advocated by some writers.

The chief change which the adoption of these equations would require in the formulas used by the electrical engineer is in calculations of magnetic fields produced by currents and of capacities of condensers from their dimensions. Here the 4π appearing frequently when e.m.u. are used generally has to be replaced by 120π (more accurately 376.735). All formulas of circuit theory, all formulas for induced e.m.f.'s remain as at present. In calculating the magnetic induction or flux density B from the magnetic intensity or field strength H, the formula $B_p = (\mu/c_p)H_p$ must be used, however, where μ is the permeability relative to vacuum. Similar remarks apply to the relation between the electric displacement or flux density D and the electric intensity or field strength E.

TRANSFORMATION OF EQUATIONS.

To transform an equation from Gaussian c.g.s. units to practical m.k.s. units or *vice-versa* the relations specified in Table I are needed.

The small number of transformation factors should be noted. This simplifying characteristic is due to the fact that both the Gaussian and the proposed practical equations are symmetrical. When changing from Gaussian units to prac-

TABLE I.

Relations between Quantities in Gaussian Units (Subscript g) and in Practical Units (Subscript p).

$$\left.\begin{matrix} l_g \\ M_g \\ t_g \\ K_g \\ F_g \\ W_g \\ \mathfrak{P}_g \end{matrix}\right\} = \left\{\begin{matrix} l_p \\ M_p \\ t_p \\ K_p \\ F_p \\ W_p \\ \mathfrak{P}_p \end{matrix}\right. \qquad \left.\begin{matrix} q_g \\ m_g \\ \rho_g \\ \tau_g \\ p_g \\ \mathfrak{M}_g \\ P_g \\ I_g \\ i_g \\ j_g \end{matrix}\right\} = \sqrt{30c} \left\{\begin{matrix} q_p \\ m_p \\ \rho_p \\ \tau_p \\ p_p \\ \mathfrak{M}_p \\ P_p \\ I_p \\ i_p \\ j_p \end{matrix}\right.$$

$$c_g = c_p$$

$$\left.\begin{matrix} E_g \\ H_g \\ \mathcal{E}_g \\ \mathcal{F}_g \end{matrix}\right\} = \frac{1}{\sqrt{30c}} \left\{\begin{matrix} E_p \\ H_p \\ \mathcal{E}_p \\ \mathcal{F}_p \end{matrix}\right. \qquad \left.\begin{matrix} D_g \\ B_g \\ \psi_g \\ \phi_g \end{matrix}\right\} = \sqrt{\frac{c}{30}} \left\{\begin{matrix} D_p \\ B_p \\ \psi_p \\ \phi_p \end{matrix}\right.$$

$$\left.\begin{matrix} R_g \\ L_g \end{matrix}\right\} = \frac{1}{30c} \left\{\begin{matrix} R_p \\ L_p \end{matrix}\right. \qquad \begin{matrix} C_g = 30cC_p \\ \mathfrak{R}_g = \frac{1}{c}\,\mathfrak{R}_p \end{matrix}$$

tical units c should be taken as c_p and when changing from practical to Gaussian as c_g. The factor 30 appearing in the table is more accurately $10\epsilon = 29.9796$.

To illustrate the derivation of the transformation factors in the table, consider first charge q. Putting the value of α_p in (17) into (16) the required relation is found at once. As $q_g E_g = q_p E_p$ the transformation for E follows immediately. Again since $D_g = \kappa E_g$, $D_p = (\kappa/c_p)\, E_p$, it is seen that

$$D_g = c_p \frac{E_g}{E_p} D_p,$$

which furnishes the transformation for D.

As an example of the use of these transformations, we have for the capacitance of a parallel plate condenser of n plates of area A and separation d in Gaussian units (e.s.u.)

$$C_g = \frac{(n-1)\kappa A_g}{4\pi d_g}. \tag{25}$$

From the table $C_g = 30cC_p$, $A_g = A_p$, $d_g = d_p$. Hence

$$C_p = \frac{(n-1)\kappa A_p}{120\pi c_p d_p}. \tag{26}$$

Again, equation (21) for Ampère's law in practical units gives for the magnetomotive force around a closed curve of length l_p linking N times a current i_p

$$H_p l_p = 120\pi N i_p$$

or

$$H_p = 120\pi \frac{N}{l_p} i_p = 120\pi n_p i_p. \tag{27}$$

This is the formula for the magnetic field inside a long solenoid of n_p turns per meter. From the table $H_p = \sqrt{30c}H_g$, $n_p = n_g$, $i_p = (1/\sqrt{30c})\, i_g$. Therefore

$$H_g = \frac{4\pi n_g}{c_g} i_g, \tag{28}$$

which differs from the familiar formula in e.m.u. because the Gaussian unit of current is the e.s.u.

UNITS AND DIMENSIONS.

The names (where a name exists), magnitudes and physical dimensions of the proposed m.k.s. practical units and of the c.g.s. Gaussian units are listed in Table II. Also the number of Gaussian units in a practical unit is given in the last column. It must be remembered that where 3 occurs as a factor in the table the more accurate value is 2.99796. All the information contained in the table is obtained, of course, from the two sets of fundamental equations already given.

It should be noted that the physical dimensions are the same and that the number of Gaussian units in a practical unit are the same for corresponding electric and magnetic quantities. This is a simplifying characteristic of symmetrical sets of fundamental equations.

To show the consistency of the figures giving the number of Gaussian units in a practical unit with the transformations in Table I, we shall deduce the one from the other for a couple of cases.

Consider, for instance, current. From Table I, $i_g = \sqrt{30c}\, i_p$. Denoting numerics as before by letters with bars over them, and taking the units from the dimensions listed in Table II,

TABLE II.

Quantity.	Symbol.	Practical m.k.s. Units.			Gaussian c.g.s. Units.			Number of Gaussian Units in a Practical Unit.
		Name.	Magnitude.	Dimensions.	Name.	Magnitude.	Dimensions.	
Length	l	meter	$(10)^2$ cm.	L	cm.	1 cm.	L	$(10)^2$
Mass	M	kilogram	$(10)^3$ gm.	M	gm.	1 gm.	M	$(10)^3$
Time	t	sec.	1 sec.	T	sec.	1 sec.	T	1
Force	K	—	$(10)^5$ dyne	MLT^{-2}	dyne	1 dyne	MLT^{-2}	$(10)^5$
Force per unit vol.	F	—	$(10)^{-1}$ dyne/cm.^{3}	$ML^{-1}T^{-2}$	—	1 dyne/cm.^{3}	$ML^{-1}T^{-2}$	$(10)^{-1}$
Energy	W	joule	$(10)^7$ erg	ML^2T^{-2}	erg	1 erg	ML^2T^{-2}	$(10)^7$
Power	$\mathcal{P}$	watt	$(10)^7$ erg/sec.	ML^2T^{-3}	erg/sec.	1 erg/sec.	ML^2T^{-3}	$(10)^7$
Charge	q	coulomb	$(10)^{-1}$ e.m.u.	$M^{1/2}LT^{-1/2}$	—	1 e.s.u.	$M^{1/2}L^{3/2}T^{-1}$	$3(10)^9$
Pole Strength	m	—	$3(10)^9$ e.m.u.		—	1 e.m.u.		
Charge/unit vol.	ρ	coul./meter^3	$(10)^{-7}$ e.m.u.	$M^{1/2}L^{-2}T^{-1/2}$	—	1 e.s.u.	$M^{1/2}L^{-1/2}T^{-1}$	$3(10)^3$
Pole/unit vol.	τ	—	$3(10)^3$ e.m.u.		—	1 e.m.u.		
Elect. Field Int.	E	volt/meter	$(10)^6$ e.m.u.	$M^{1/2}T^{-3/2}$	—	1 e.s.u.	$M^{1/2}L^{-1/2}T^{-1}$	$\frac{1}{3}(10)^{-4}$
Mag. Field Int.	H	—	$\frac{1}{3}(10)^{-4}$ e.m.u.		gauss	1 e.m.u.		
Elect. Displacement	D	—	$\frac{1}{12\pi}(10)^{-5}$ e.m.u.	$M^{1/2}L^{-1}T^{-1/2}$	—	$\frac{1}{4\pi}$ e.s.u.	$M^{1/2}L^{-1/2}T^{-1}$	$(10)^4$
Mag. Induction	B	—	$(10)^4$ e.m.u.		gauss	1 e.m.u.		

TABLE II—(*Continued*).

Quantity.	Symbol.	Practical m.k.s. Units.			Gaussian c.g.s. Units.			Number of Gaussian Units in a Practical Unit.
		Name.	Magnitude.	Dimensions.	Name.	Magnitude.	Dimensions.	
Elect. Flux	ψ	—	$\frac{1}{12\pi}(10)^{-2}$ e.m.u.	$M^{1/2}LT^{-1/2}$	—	$\frac{1}{4\pi}$ e.s.u.	$M^{1/2}L^{3/2}T^{-1}$	$(10)^8$
Mag. Flux	ϕ	—	$(10)^8$ e.m.u.		—	1 e.m.u.		
e.m.f. or Elect. Pot	$\mathcal{E}$	volt	$(10)^8$ e.m.u.	$M^{1/2}LT^{-2/2}$	—	1 e.s.u.	$M^{1/2}L^{1/2}T^{-1}$	$\frac{1}{3}(10)^{-2}$
m.m.f. or Mag. Pot	$\mathcal{F}$	—	$\frac{1}{3}(10)^{-2}$ e.m.u.		—	1 e.m.u.		
Elect. Moment	$\mathfrak{p}$	—	10 e.m.u.	$M^{1/2}L^3T^{-1/2}$	—	1 e.s.u.	$M^{1/2}L^{5/2}T^{-1}$	$3(10)^{11}$
Mag. Moment	$\mathfrak{M}$	—	$3(10)^{11}$ e.m.u.		—	1 e.m.u.		
Polarization	P	—	$(10)^{-5}$ e.m.u.	$M^{1/2}L^{-1}T^{-1/2}$	—	1 e.s.u.	$M^{1/2}L^{-1/2}T^{-1}$	$3(10)^5$
Magnetization	I	—	$3(10)^6$ e.m.u.		—	1 e.m.u.		
Current	i	ampere	$(10)^{-1}$ e.m.u.	$M^{1/2}LT^{-2/2}$	—	1 e.s.u.	$M^{1/2}L^{3/2}T^{-2}$	$3(10)^9$
Current Density	j	amp./meter^2	$(10)^{-5}$ e.m.u.	$M^{1/2}L^{-1}T^{-2/2}$	—	1 e.s.u.	$M^{1/2}L^{-1/2}T^{-2}$	$3(10)^5$
Resistance	R	ohm	$(10)^9$ e.m.u.	O	—	1 e.s.u.	$L^{-1}T$	$\frac{1}{9}(10)^{-11}$
Capacitance	C	farad	$(10)^{-9}$ e.m.u.	T	—	1 e.s.u.	L	$9(10)^{11}$
Inductance	L	henry	$(10)^9$ e.m.u.	T	—	1 e.s.u.	$L^{-1}T^2$	$\frac{1}{9}(10)^{-11}$
Reluctance	$\mathcal{R}$	—	$\frac{1}{3}(10)^{-10}$ e.m.u.	T^{-1}	—	1 e.m.u.	L^{-1}	$\frac{1}{3}(10)^{-10}$

$$\bar{i}_g \frac{\text{gm.}^{1/2}\,\text{cm.}^{3/2}}{\text{sec.}^2} = 3(10)^{9/2} \frac{\text{meter}^{1/2}}{\text{sec.}^{1/2}} \bar{i}_p \frac{\text{kg.}^{1/2}\,\text{meter}}{\text{sec.}^{3/2}}$$

$$= 3(10)^9 \bar{i}_p \frac{\text{gm.}^{1/2}\,\text{cm.}^{3/2}}{\text{sec.}^2}.$$

Therefore $\bar{i}_g = 3(10)^9 \bar{i}_p$ and there are $3(10)^9$ Gaussian c.g.s. units (e.s.u.) of current in a practical m.k.s. unit (ampere).

Next consider magnetic induction or flux density B. We have

$$\bar{B}_g \frac{\text{gm.}^{1/2}}{\text{cm.}^{1/2}\,\text{sec.}} = (10)^{7/2} \frac{\text{meter}^{1/2}}{\text{sec.}^{1/2}} \bar{B}_p \frac{\text{kg.}^{1/2}}{\text{meter sec.}^{1/2}}$$

$$= (10)^4 \bar{B}_p \frac{\text{gm.}^{1/2}}{\text{cm.}^{1/2}\,\text{sec.}},$$

whence $\bar{B}_g = (10)^4\ \bar{B}_p$ and consequently there are $(10)^4$ Gaussian c.g.s. units (gauss) in a practical m.k.s. unit.

SIMPLE EXAMPLES OF NUMERICAL CALCULATIONS IN PRACTICAL M.K.S. UNITS.

Example 1. A point charge of $(10)^{-4}$ coulomb is immersed in a dielectric of permittivity $\kappa = 2$. Find the electric flux and D and E at a distance of 10 meters.

We will deduce the required equations in practical units directly from the electromagnetic equations (18)–(24). From (18) the electric flux through a sphere of radius r described about the charge is

$$\psi_p \equiv 4\pi r_p^2 D_p = 120\pi q_p = 12\pi(10)^{-3} \text{ line}$$

and hence the displacement or flux density is

$$D_p \equiv \frac{\psi_p}{4\pi r_p^2} = 30\,\frac{q_p}{r_p^2} = 30\,\frac{(10)^{-4}}{(10)^2} = 3(10)^{-5} \text{ line/meter}^2.$$

Using (22), the electric field strength is

$$E_p = \frac{c_p}{\kappa} D_p = 30\,\frac{c_p q_p}{\kappa r_p^2} = 30\,\frac{3(10)^8 \cdot (10)^{-4}}{2 \cdot (10)^2}$$

$$= \frac{9}{2}(10)^3 \text{ volt/meter}.$$

Example 2. A long straight solenoid of $(10)^{-1}$ meter^2 cross-section and $(10)^4$ turns per meter is wound on a core of permeability $\mu = (10)^2$. Find H and B when a current of 10 amp. is flowing through it. If a secondary of one turn surrounds the solenoid, find the mutual inductance and the electromotive force when the current changes at the rate of 100 amp./sec.

Formula (27) for H_p gives

$$H_p = 120\pi n_p i_p = 120\pi(10)^4 \cdot 10 = 12\pi(10)^6 \text{ practical units.}$$

$$B_p = \frac{\mu}{c_p} H_p = \frac{(10)^2}{3(10)^8} 12\pi(10)^6 = 4\pi \text{ line/meter}^2.$$

$$\text{Flux } \phi_p = (10)^{-1} B_p = 4\pi(10)^{-1} \text{ line.}$$

$$\text{Mutual Inductance } M_p = \frac{\phi_p}{i_p} = 4\pi(10)^{-2} \text{ henry.}$$

$$\text{Electromotive Force } \varepsilon_p = -M_p \frac{di_p}{dt} = -4\pi(10)^{-2} \cdot (10)^2$$

$$= -4\pi \text{ volt.}$$

Example 3. A parallel plate condenser is built of 101 plates of area $(10)^{-1}$ meter^2 each, $(10)^{-3}$ meter apart. If the dielectric has a permittivity $\kappa = 5$, find its capacitance. What charge Q_p will it acquire when raised to a potential V_p of 100 volts?

Formula (26) for C_p gives

$$C_p = \frac{(n-1)\kappa A_p}{120\pi c_p d_p} = \frac{(10)^2 \cdot 5 \cdot (10)^{-1}}{120\pi \cdot 3(10)^8 \cdot (10)^{-3}}$$

$$= \frac{5}{36\pi} (10)^{-5} \text{ farad.}$$

$$Q_p = C_p V_p = \frac{5}{36\pi} (10)^{-5} \cdot (10)^2 = \frac{5}{36\pi} (10)^{-3} \text{ coulomb.}$$

Example 4. A magnet of moment $(10)^{-6}$ practical m.k.s. unit is moving away from a circular turn of wire of area $(10)^{-4}$ meter^2 along the axis of the circle with velocity of 10 meter/sec., the axis of the magnet being perpendicular to the plane of

the turn of wire. Find the induced e.m.f. in the wire when the magnet is at a distance of 2 meters.

The magnetic field due to a magnet of moment $\mathfrak{M}$ at a point on its axis is

$$H_p = \frac{60c_p\mathfrak{M}_p}{\mu r_p^3},$$

and

$$B_p = \frac{\mu}{c_p} H_p = \frac{60\mathfrak{M}_p}{r_p^3}.$$

Hence

$$\frac{dB_p}{dt} = -\frac{180\mathfrak{M}_p}{r_p^4}\frac{dr_p}{dt} = -\frac{180(10)^{-6}}{(2)^4}(10) = -\frac{9}{8}(10)^{-4}$$

practical unit, and

$$\mathcal{E}_p = -A_p\frac{dB_p}{dt} = (10)^{-4}\cdot\frac{9}{8}(10)^{-4} = \frac{9}{8}(10)^{-8} \text{ volt.}$$

IMPORTANT DERIVED FORMULAS.

	Practical m.k.s.	*Gaussian c.g.s.*
Force between two charges	$K_p = \frac{30c_pq_pq_p'}{\kappa r_p^2}$,	$K_g = \frac{q_gq_g'}{\kappa r_g^2}$,
Force between two poles	$K_p = \frac{30c_pm_pm_p'}{\mu r_p^2}$,	$K_g = \frac{m_gm_g'}{\mu r_g^2}$,
Electric potential due to point charge	$V_p = \frac{30c_pq_p}{\kappa r_p}$,	$V_g = \frac{q_g}{\kappa r_g}$,
Magnetic potential due to point pole	$\Omega_p = \frac{30c_pm_p}{\mu r_p}$,	$\Omega_g = \frac{m_g}{\mu r_g}$,
Capacity of parallel plate condenser	$C_p = \frac{\kappa A_p}{120\pi c_p d_p}$,	$C_g = \frac{\kappa A_g}{4\pi d_g}$,
Field between charged plates (σ = charge per unit area)	$E_p = \frac{120\pi c_p\sigma_p}{\kappa}$,	$E_g = \frac{4\pi\sigma_g}{\kappa}$,
Magnetic field in solenoid	$H_p = 120\pi n_p i_p$,	$H_g = \frac{4\pi n_g}{c_g} i_g$.

Force on current in magnetic field	$K_p = B_p i_p l_p,$	$K_g = \frac{B_g i_g l_g}{c_g},$
Magnetomotive force around closed curve encircling current N times	$\mathcal{F}_p = 120\pi N i_p,$	$\mathcal{F}_g = \frac{4\pi N i_g}{c_g},$
Energy per unit volume in electric field	$w_p = \frac{\kappa E_p^2}{240\pi c_p},$	$w_g = \frac{\kappa E_g^2}{8\pi},$
Energy per unit volume in magnetic field	$w_p = \frac{\mu H_p^2}{240\pi c_p},$	$w_g = \frac{\mu H_g^2}{8\pi},$
Energy in charged condenser	$W_p = \frac{1}{2} C_p V_p^2,$	$W_g = \frac{1}{2} C_g V_g^2,$
Energy in inductance	$W_p = \frac{1}{2} L_p i_p^2,$	$W_g = \frac{1}{2} L_g i_g^2.$

All equations of circuit theory (including Ohm's law) remain as at present when expressed in either Gaussian c.g.s. or practical m.k.s. units, and all equations for induced e.m.f.'s in terms of rate of change of flux remain as at present when expressed in practical m.k.s. units.

The factor 30 appearing in some of the practical equations listed above is more accurately $10\epsilon = 29.9796$ and 120π is more accurately $40\pi\epsilon = 376.735$.

Non-Sparking Tools.—(*Chemical Industries*, Vol. 35, p. 45.) A new line of non-sparking tools has recently appeared on the market. Such tools should find frequent application in lacquering plants, by producers and processors of cellulose nitrate, oil companies and all other industries confronted with explosion and fire hazards in their production and maintenance work. These new tools are made of wrought beryllium copper; are non-magnetic and non-sparking and by actual test are almost as durable as steel tools of similar design and size.

C.

Prussic Acid Antidote.—(U. S. D. A. Clip Sheet, No. 842.) Three veterinary scientists of the Bureau of Animal Industry, A. B. CLAWSON, H. BUNYEA, and J. F. COUCH, have been testing various chemicals as remedies for prussic-acid poisoning. They found that methylene blue, sodium tetrathionate, sodium nitrite, sodium thiosulphate were effective and the two latter in combination being especially so.

Hydrocyanic acid does not occur in dangerous quantities in healthy growing plants but does develop in many valuable forage plants when normal growth has been retarded or stopped by drought, frost, bruising, trampling, wilting, mowing or other cause. The antidotes listed, if administered in time, will save the lives of animals poisoned by eating such plants containing appreciable quantities of hydrocyanic or prussic acid. In the experiments, the first of their kind conducted under field conditions, the animals were drenched with doses of potassium cyanide in water ranging from a lethal dose to 2½ times the lethal dose. The remedies were administered at various times after the cyanide was given.

C.

THE WASHINGTON MONUMENT.

Is It Earthquake Proof?
Is Spalling Preventable?

BY

JACOB J. CRESKOFF,

Structural Engineer, Member of the Institute.

DESCRIPTION.

The Washington Monument stands several hundred feet to the southeast of the point of intersection of the north-south axis through the White House and the east-west axis through the Capitol. The site was chosen originally because it was believed to be a bed of closely compacted sand and gravel. However, after construction had been completed, it was found that the monument was settling at the average rate of one-fortieth of an inch annually. Obviously there was some soft material under the sand and gravel deposit.

Borings taken in 1931 disclosed that layers of sand, gravel, and clay, or mixtures thereof, were to be found between the foundation of the monument and bedrock. In the 4–20–33 issue of Engineering News-Record, Major Gillette states that "the deposit of clay overlying the rock in the vicinity of the Washington Monument is in such soft and moist condition as to be liable to flow under any considerable shifting of its superimposed load." It is to be expected, therefore, that during an earthquake the site will yield to the vibrations of the monument.

The substructure consists of a spread footing 126′ $5\frac{1}{2}$″ square and 36′ 10″ deep, the top of the foundation coinciding with the existing profile. The original foundation was constructed of blue gneiss laid in pure lime mortar. Later, the foundation was increased in area and depth by underpinning with concrete.

The superstructure of the monument is square in section,

and was constructed of solid masonry walls. It consists of a shaft 500′ 5″ tall, surmounted by a 55′ 0″ pyramidion. The total height of the superstructure is therefore 555′ 5″. From the top of the foundation to the 150′ level, the facing is white marble, with a Potomac blue stone backing; both laid in lime putty mortar. The interstices are filled with mortar and spalls. The walls between these limits vary in thickness from 15′ 0″ at the foundation to 11′ 10¼″ at the 150′ level. The weight of this part of the shaft is 22,373 tons.

From the 150′ level to the pyramidion, a distance of 350′ 5″, the masonry consists of ashlar, or cut stones, marble faced and granite backed, alternate header and stretcher. When the walls become sufficiently thin for the stretchers to show, the granite backing drops off, and thereafter the walls are entirely of marble. Between these limits the stones are laid in Portland cement mortar. The wall thickness varies from 11′ 10¼″ at the 150′ level to 1′ 6″ at the pyramidion. This part of the shaft weighs 21,260 tons.

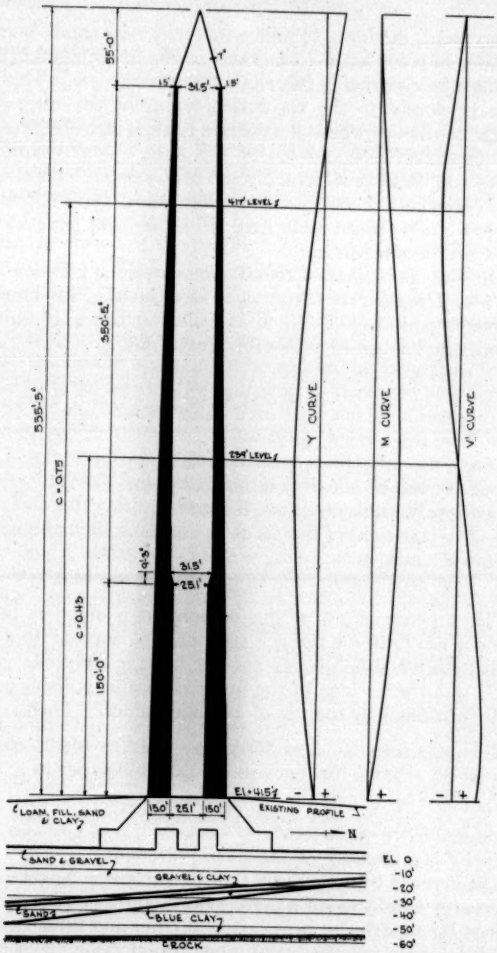
The pyramidion is constructed of 7″ thick marble slabs. These rest upon corbels on ribs, there being three ribs to a side. No mortar was used here, the waterproofing being accomplished by means of copper strips in the joints. The pyramidion weighs 300 tons.

The metal work in the superstructure weighs 275 tons. It consists of eight built-up wrought iron columns extending from the foundation into the pyramidion, and the framing for the elevator shaft, the stairways, and the landings. The metal work is not shown in the accompanying sketch because, data regarding its layout and sectional area not being available, its influence was neglected in the investigation of the various sections.

The total weight of the superstructure is therefore 44,208 tons, or 88,416,000 lbs. A vertical section through the monument and the site is shown in the sketch.

ANALYSIS.

The free periods of transverse vibration of the Washington Monument, as measured by F. Neumann, Chief, Seismology Section, U. S. Coast and Geodetic Survey, are: 1.81 sec., fundamental, and 0.60 sec., first harmonic. The horizontal



SECTION THRU MONUMENT
& SITE
SCALE: 1"=50'

displacement, during a 30 mile wind, was very small, being of the order of 0.002 inch. The ratio of the fundamental to the first harmonic period is therefore 1.81/0.60 = 3.0. This corresponds closely to the theoretical value of the ratio for a vertical cantilever whose foundation is fixed into compressible soil. (See Par. 6.8 and 9.9 of the author's "Dynamics of Earthquake Resistant Structures," McGraw-Hill Book Company, New York.) Since the borings show the existence of a layer of soft, moist blue clay, it seems that practice and theory are in agreement.

The end conditions of the above mentioned cantilever are known as *Hinged-Free*. For this case, the free, fundamental transverse period of vibration of a slender beam, of uniform section, and composed of isotropic material, is

$$T = 0.41 \sqrt{\frac{wL^4}{EIg}},$$

where T = free period, sec.
w = weight, lbs. per inch of L.
L = height above point of fixity, in.
E = Young's modulus, lbs. per sq. in.
I = moment of inertia of the section about its neutral axis, in.^{4}
g = acceleration due to gravity, in. per sec.^{2}

The superstructure of the monument is slender, but its section is not uniform, nor is it composed of isotropic material. A quantitative solution has not yet been attempted. However, a qualitative idea of the induced seismic stresses can be readily obtained by the use of average values. Thus,

T = 1.81 sec., L = 6665 in., $L^4 = 197 \times 10^{13}$,
w = 88,416,000 lbs./6665 in. = 13,250 lbs. per in.,
g = 386 in. per sec.^{2}

Therefore, $EI = 348 \times 10^{13}$,
where EI is a measure of the elastic resistance.

The layer of blue clay is not at the surface. Nevertheless, the greater amplitude of a more mobile soil is not appreciably reduced by superincumbent layers of drier and more compact materials, such as the sand and gravel beds. Assuming, therefore, a surface amplitude of 2″ for the blue clay, the am-

plitude assigned to the foundation during a major earthquake becomes $F = 0.93''$. (See Par. 3.9, 9.5, and 9.6 of D. of E. R. S.).

A preponderance of opinion indicates that destructive earthquakes occur in a bracket of periods ranging from 1.0 to 1.5 seconds. Of course, there is no assurance that any assumed earthquake period T_p will be correct. However, the gravest stresses occur as the value of T_p approaches that of T, i.e., as the period of the earthquake synchronizes with that of the structure. The author believes that the worst case which need be considered practically will be covered by assuming a ratio of $T/T_p = 0.9$ or 1.1. His reasoning is as follows: that complete synchronism is highly improbable, but, if it should occur, the factor of safety inherent in the structure will be ample to take care of the temporary, increased stresses provided that the structure has been designed for a ratio of $T/T_p = 0.9$ or 1.1.

Therefore, although for ordinary structures the method of arriving at T/T_p by comparing T with the dangerous bracket of periods will suffice, nevertheless, for monumental structures it is recommended that a ratio of $T/T_p = 0.9$ or 1.1 be used. Accordingly, although $T/T_p = 1.81/1.50 = 1.21$, a ratio of $T/T_p = 1.1$ will be adopted.

The earthquake chosen for the purpose of the analysis is consequently defined, for continuous motion, as imparting an amplitude of $F = 0.93''$ to the foundation, and of such period T_p as to give a ratio of $T/T_p = 1.1$. The following table was compiled by means of this ratio. (See Fig. 12 of D. of E. R. S.)

c	Y_u	M_u	V_u
0.0	1.00	0.00	2.16
0.2	−1.29	1.85	1.99
0.4	−2.38	2.88	0.36
0.6	−1.59	2.43	−1.43
0.8	0.74	0.92	−1.85
1.0	3.83	0.00	0.00

where $c = x/L$,
x = distance from bottom,
Y_u = maximum unit deflection,
M_u = maximum unit moment,
V_u = maximum unit shear.

For the *hinged-free* case, $m'L = 3.93\ (T/T_p)^{\frac{1}{2}}$ where m' is a constant of forced vibration. (See Eq. 96, and 138 of D. of E. R. S.). Therefore,

$$m'L = 3.93 \times 1.05 = 4.13, \quad m' = 4.13/6665 = 6.2 \times 10^{-4},$$
$$m'^2 = 38.4 \times 10^{-8}, \quad m'^3 = 238 \times 10^{-12},$$

$$EIm'^2F = 348 \times 10^{13} \times 38.4 \times 10^{-8} \times 0.93 = 124 \times 10^7,$$
$$EIm'^3F = 348 \times 10^{13} \times 238 \times 10^{-12} \times 0.93 = 771 \times 10^3.$$

Also (see Eq. 180, 181, and 182 of D. of E. R. S.),

$$y = FY_u, \qquad M = EIm'^2FM_u, \qquad V = EIm'^3FV_u.$$

The actual maximum seismic deflections Y, moments M, and shears V, are shown in the following table.

c	$\frac{y}{''}$	$\frac{Y}{''}$	$\frac{M}{''\#}$	$\frac{V}{\#}$	$\frac{V'}{\#}$
0.0	0.93	0.00	0	167×10^4	334×10^4
0.2	−1.20	−2.13	230×10^7	153×10^4	306×10^4
0.4	−2.22	−3.15	358×10^7	28×10^4	56×10^4
0.6	−1.48	−2.41	302×10^7	-110×10^4	-220×10^4
0.8	0.69	−0.24	114×10^7	-143×10^4	-286×10^4
1.0	3.56	2.63	0	0	0

where y = deflection relative to the space axis,
Y = deflection relative to the foundation,
V' = static equivalent of the seismic shear.

The Y, M, and V' curves are shown in the sketch. The maximum deflection, 3.2″, occurs where $c = 0.43$, or at the 239′ level. Points of maximum shear occur at the bottom, where $V' = 334 \times 10^4$ lbs., and where $c = 0.75$, or at the 417′ level, where $V' = 290 \times 10^4$ lbs. The maximum moment occurs at the 239′ level, and equals 360×10^7 in.-lbs.

Investigating the section at the 417′ level,

$$M = 170 \times 10^7 \text{ in.-lbs.} \qquad V' = 290 \times 10^4 \text{ lbs.}$$

From scaled dimensions, the effective area of the section is $A = 65{,}000$ in.^{2} The moment of inertia is $I = 19 \times 10^8$ in.^{4} From the neutral axis to the extreme fiber is $c = 228$ in. The weight above the section is estimated $P = 474 \times 10^4$ lbs.

The compressive and tensile stresses, computed from the formula $= s\ (P/A) \pm (Mc/I)$, are:

$$s_c = 277 \text{ lbs. per in.}^2. \qquad s_t = 131 \text{ lbs. per in.}^2$$

where the subscripts c and t denote compressive and tensile stresses respectively. The unit shear is $v = 45$ lbs. per in.^{2}

Investigating the section at the 239′ level,

$M = 360 \times 10^7$ in.-lbs. $\quad V' = 0 \quad A = 144{,}000$ in.^{2}
$I = 527 \times 10^7$ in.^{4} $\quad c = 270$ in. $\quad P = 244 \times 10^5$ lbs.

Therefore, $s_c = 354$ lbs. per in.^{2} $\quad s_t = 14$ lbs. per in.^{2}

Investigating the section at the bottom,

$M = 0 \quad V' = 334 \times 10^4$ lbs. $\quad A = 336{,}000$ in.^{2}
$P = 884 \times 10^5$ lbs.

Therefore, $\quad s_c = 263$ lbs. per in.^{2} $\quad v = 10$ lbs. per in.^{2}

CONCLUSIONS.

With the exception of the vicinity of the 417′ level, the unit stresses are all within the allowable values for the stone and mortar. At the 417′ level, the unit tensional stress approaches the ultimate value for Portland cement mortar in tension. But, if the effect of the structural iron were taken into account, the theoretical tensional stress would be less.

Subject to the results shown by a quantitative analysis, the author is of the opinion that, under existing conditions, the Washington Monument is safe on its present site; that if the shaft should be damaged during an earthquake, failure will occur near the 417′ level; that settlement may speed up during major earthquakes, but the monument will survive without serious damage.

It may be of interest to know that a moderate earthquake which occurred in Ohio and Indiana on September 19, 1884, was felt by and alarmed workmen on the 500′ level of the unfinished monument.

IS SPALLING PREVENTABLE?

The surface of the Washington Monument has been continuously marred by progressive spalling of the masonry edges

in the lower 150′. This phenomenon has been attributed to static overstressing of the masonry joints. It is now proposed to repair the existing damage and to prevent future spalling by cutting back on the joints and repointing with some suitable material.

The author disagrees with the diagnosis of the cause, and the efficacy of the treatment. In his opinion, spalling is a dynamic phenomenon due to the mode of vibration of the monument, as well as to the nature and dimensions of the materials of construction.

It must be kept in mind that the monument is in a constant state of vibration, principally because of wind gust action. Again, its deflection curve contains sharp slope changes over the lower part by reason of the hinged-free end condition. Finally, very little, if any, spalling occurs above the 150′ level, where the masonry thins abruptly.

Evaluating these factors, it becomes evident that the rigid blocks in the lower 150′ fail because they cannot withstand the continuous bending reversals of vibration, the steady alternating push and pull of the mortar on the edges of the rigid blocks eventually causing the spalling. On the other hand, the relatively flexible masonry slabs above, by reason of being better able to withstand the bending reversals should, and do, fare much better.

The author therefore predicts that the proposed treatment to end spalling will fail of its purpose because it ignores that the spalling is a dynamic phenomenon. Sooner or later, and for the same reasons which caused the previous failure, spalling will recur.

For a complete cure, reconstruction of the lower 150′ of the monument is indicated. A simpler solution, however, is available.

Admitting that vibration cannot be prevented, and that spalling will continue but is immaterial insofar as structural strength is concerned, the problem resolves itself into making the monument look well to the general public. Hence, why not cover up all evidence of spalling?

One way of doing this would be to cover the monument with an ornamental metal, say stainless steel. Or, if the Art Jury objects to this method on æsthetic grounds, a veneer of

marble—fastened in sections so that alignment or replacement is easy—might be hung on the lower 150′ of the shaft. Spalling of the surface proper would, of course, continue, but would be invisible.

In justice to the men in charge of the Washington Monument, it must be stated that the cutting back and repointing method of repair was adopted principally because available funds did not permit the more comprehensive scheme. Nevertheless, it seems a pity, and inconsistent with true economy, that a permanent remedy could not be undertaken at this time.

The author is indebted to F. Neumann, Chief, Seismology Section, U. S. Coast and Geodetic Survey; to C. A. Peters, Chief, Designing Section, and J. L. Nagle, Assistant Chief of the Division, U. S. National Parks, Buildings, and Reservations; and to P. H. Bates, Chief, Masonry Research, U. S. Bureau of Standards, for their interested coöperation in furnishing required data.

A Peace Task for Big Berthas.—(U. S. D. A. Clip Sheet No. 843.) Meteorologists of the Weather Bureau believe that the Big Berthas of World War fame hold promise as valuable peace time weapons for scientists interested in upper air exploration. In the bombardment of Paris nearly 20 years ago these guns hurled projectiles 24 miles into the air while the present limit for meteorological scouting balloons is only 22 miles.

Recording instruments carried aloft in kites, balloons, or airplanes have cleared up much of the mystery of the lower air, the so-called troposphere, and of the lower levels of the stratosphere, just above the troposphere. Observations in the troposphere show that air here moves in many ascending and descending currents, causing the formation and dissipation of clouds.

Air movements in the stratosphere on the other hand are believed to be generally horizontal. Although this upper region is usually free from clouds, the so-called nacreous and noctilucent clouds are occasionally seen there. Their nature and origin will be cleared up only after further exploration.

Another interesting field of stratosphere investigation is the layer of ozone between 22 and 37 miles above the earth. Biologically this layer is of tremendous importance because it absorbs solar rays that would destroy life on the earth if permitted to penetrate that far. This absorption of radiant energy is believed, though not proved, to raise the temperature 50 degrees or more above that on the earth in summer.

C.

DESIGN AND TEST OF A HIGH-PERFORMANCE SERVO-MECHANISM.

BY

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INTRODUCTION.

In a companion paper * attention is given to the analysis of the performance of servo-mechanisms of various types in terms of their mode of action and their parameters. In this paper the point of view changes from the analyses of given devices to the design of one device for a given performance or for the best possible performance. Whereas three principal types of servo-mechanism were analyzed in the first paper, consideration in this paper is somewhat arbitrarily but rationally limited to one, the continuous-control type.

Before proceeding further it may be well to restate what is meant by a servo-mechanism or, for brevity, a servo. A servo-mechanism is a power-amplifying device in which the output indications are made to reproduce as nearly as practicable the input indications by applying a driving force to the output depending on the deviation of the output from the input, and in which this driving force is sufficiently large to drive associated equipment such as recording or control apparatus. Such a device is also called a "follow-up" mechanism.

The occasion for the design developed here was the need for an automatic balancing and recording device to be used in a calculating machine in which light flux is the measure of an integral. In this device are two photoelectric cells each receiving components of light flux which vary with time. The servo is so to control an additional component entering one cell that the total light flux entering this cell is the same as that entering the other. The magnitude of this additional

* Hazen, H. L., Theory of Servo-Mechanism, J. F. I., Vol. 218, No. 3, Sept. 1934, pp. 279–331.

component is also to be recorded graphically. The servo-mechanism is to utilize the difference between the drops in potential in the two photoelectric cells as its input. Its output operates a light shutter by which the additional component of light flux is controlled, and also operates the recording mechanism.

While the design of this servo is built around a particular application, the significant elements entering into the design are common to many servo applications. Consequently, although definiteness is secured by considering a specific numerical case in detail, the method of attack illustrated is rather general and can be applied in a wide variety of cases.

QUALITATIVE DESIGN.

In what follows, the plan of procedure is first, to lay out a design qualitatively, considering the limitations fixed by properties of materials, and the special conditions in which the apparatus is to be used; second, to formulate a quantitative criterion of merit for the design; and third, to develop a particular design in which all factors contributing to a meritorious design are given limiting values. These three aspects will be considered in order, beginning with the general requirements and their effect upon the design layout.

The first requirement is high amplification. The input to the servo is the potential difference set up between two points in a photoelectric cell circuit caused by the difference between the total light flux entering one photoelectric cell and that entering the other. The photo cells are capable of developing a useful potential difference only when working into a very high resistance so that the increment of power to which the servo input must respond is very small; in this case, it is of the order of 10^{-10} watts. On the other hand, the output must be capable of delivering a power of the order of 10 to 100 watts in response to this input variation, for operating a light shutter to eliminate the light flux unbalance, the recording mechanism, etc. Consequently an overall power amplification of 10^{11} or 10^{12} is required.

A second requirement is a non-oscillatory response of the output to avoid any possibility of resonance with periodic components in the input.

In addition, the servo output is to follow as rapidly and accurately as practicable all changes in the input.

As stated above the input to the servo is electrical in nature and suitable for operating a vacuum-tube amplifier. The output is to be a mechanical motion, most conveniently a rotation. Therefore the servo must furnish the required power amplification and convert electrical energy into mechanical energy. This implies an electrical power amplifier, an electric motor and perhaps a mechanical power amplifier. Whether or not mechanical amplification should be used depends upon considerations that will be treated presently.

Before determining the desirability of using mechanical amplification, it is well to consider what elements will be important in the dynamic response of the servo. First, what are the dynamic elements introduced by the electric amplifier? This of course depends upon the type of amplifier. Since periodicity is to be avoided in this servo, the d-c. type is preferable to the a-c., even though the latter may be easier to build. If resistance coupling is used, the action of such an amplifier is substantially independent of the time derivatives of the input at the values of these derivatives encountered in this servo application. Consequently the amplifier may be assumed to introduce no friction or inertia effects, and to affect directly only the resilience constant, i.e., the torque on the output element per unit of output-input deviation. Indirectly it affects the damping torque on the output element in a manner considered subsequently.

Next consider the dynamic elements introduced by the electric motor. These are primarily inertia and damping. Dynamically then the system including the electric amplifier and motor consists of an input, varying with time, that applies a driving torque to a body which has inertia and a retarding torque due to its velocity.

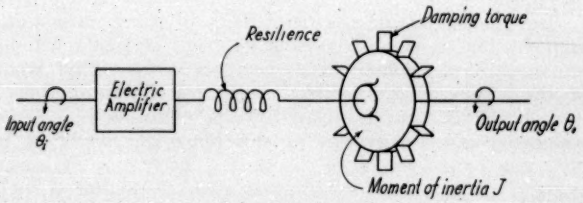
If a mechanical torque or power amplifier to step up the power output of the motor is added to this system, additional dynamic parameters will of course be introduced. However, analysis and tests have shown that the type of torque amplifier [1, 2] used in this servo introduces primarily only inertia and amplification. The inertia of this amplifier can be repre-

[1] For numbered references see bibliography.

sented as a single mass concentrated at the torque amplifier input which can be added to the inertia of the motor. Similarly, if the load that the torque amplifier drives has friction or inertia parameters, these parameters, when reduced by the amplification factor of the torque amplifier, can be considered as applied directly to the motor shaft.

It is seen then that a servo-mechanism of the type described reduces to a very simple dynamical system which is represented schematically in Fig. 1. Here the input is shown

FIG. 1.



Schematic diagram of the significant dynamic elements of the servo-mechanism considered in this paper.

in terms of angle. It will be recalled that the servo output controls light, which in turn alters the photocell current. Hence it is possible to think entirely in terms of one type of variable such as angle because, at least for small variations in all of the quantities mentioned, the variations are linearly related. Thus the apparent anomaly of an electric amplifier interposed between two rotating shafts disappears. The amplifier is shown in this position to represent correctly the fact that the reaction of the resilience element on the input is less than its reaction on the output element by the amplification introduced between input and output.

The second aspect of the design will now be considered, that of formulating a quantitative criterion of merit. In the companion paper the servo-mechanism shown in Fig. 1 is analyzed in considerable detail, and a figure of merit is developed which will serve as the basis for the design given in this paper. Briefly this figure of merit is taken as the product of two factors. One factor is the maximum speed at which the servo output will operate. This factor attaches

importance to the maximum rate of change of input which the servo can follow. The second factor is the reciprocal of the steady-state lag error or output-input deviation, expressed as a fraction of the output speed at which this error occurs. This factor attaches importance to the accuracy of following. In making up this figure of merit it is assumed that the degree of oscillation permitted in the response of the servo has been more or less definitely fixed by the nature of the servo application and that a quantity designated by γ^2, which is a measure of this degree of oscillation, has been given a definite value.

To express these facts mathematically * let

M = figure of merit.

ω_m = maximum angular velocity of servo,

θ_m = steady-state output-input deviation or lag error at output speed ω_m (θ is a linear function of ω in the steady state),

whence

$$M = \frac{\omega^2_m}{\theta_m}. \tag{1}$$

It can be shown * readily from this definition that in terms of design quantities

$$M = \frac{\tau_m}{4\gamma^2 J}, \tag{2}$$

where

τ_m = maximum permissible driving torque on output as limited by heating, stress, etc.,

J = moment of inertia of servo and load referred to servo output,

γ^2 = factor characterizing the oscillatory tendency of the output.

C.g.s. units are used here for these quantities although as no empirical factors appear, any consistent set is equally suitable.

It may be noted here that although this servo must be designed for aperiodic response, i.e., for $\gamma^2 = 1$, no complica-

* See companion paper.

tion is introduced by retaining γ^2 throughout the design thus making the results more general. Consequently this will be done.

Equation (2) is a particularly important relation, as it gives the standard for design. Stated in words it is this, that the ratio of maximum permissible applied driving torque to the servo inertia shall be a maximum for the best servo. The permissible torque is limited by two factors, heating, and available damping, in a manner discussed in detail subsequently. Any means of improving this ratio of torque to inertia improves the performance of the servo as judged by speed and accuracy.

In the companion paper it is shown that, by making one component of the driving torque applied to the output a linear function of the second time derivative of the input-output deviation, the apparent inertia can be made arbitrarily small. Also by making another component of the driving torque a linear function of the first time derivative of the deviation, and omitting damping associated with the absolute velocity of the output, the lag error can be made arbitrarily small. In the present design these two methods of improving servo performance are not used as the driving torque is made proportional only to the input-output deviation. However, the performance of any servo designed on the latter basis can be improved by the incorporation of the two additional components of driving torque.

The criterion of design then is the ratio of maximum driving torque to inertia and the problem here considered is that of making this ratio a maximum. Because of its effect upon this ratio it is appropriate at this point to consider the power level at which the transformation from electrical to mechanical power should be made, that is, whether or not a small electric motor followed by a mechanical power amplifier is preferable to a large electric motor. The key consideration is the amount of inertia associated with the production or transmission of a given torque. A brief general discussion of this subject will therefore be given.

Electromagnetically, torque is produced by current-carrying conductors placed in a magnetic field. Both the field strength and the current carrying capacity of conductors

are subject to rather definite limits. Consequently the force which can be developed electromagnetically in a unit mass of conductor is rather definitely limited. Numerically such an approximate figure is readily obtained. Thus by using the following values as representative for a copper conductor in an air gap

Flux density $\beta = 7500$ gausses (48,400 lines per $\overline{\text{in.}}^2$),
Current density $\sigma = 310$ amp. per $\overline{\text{cm.}}^2$ (approx. 2000 amp. per square inch),
Density of copper $= 8.9$,

it is found that a force of

$$\frac{7500 \times 310}{10 \times 8.9} = 2.61 \times 10^4 \text{ dynes per gm.}$$

can be developed electromagnetically.

On the other hand consider the stress that can be carried by a unit mass of steel of one cm. length. If the working stress and density of steel are

Working stress $= 6.90 \times 10^8$ dynes per $\overline{\text{cm.}}^2$ (10,000 lb. per $\overline{\text{in.}}^2$),
Density of steel $= 7.7$,

the force transmitted per gm. per cm. of length of steel is

$$\frac{6.90 \times 10^8}{7.7} = 8.96 \times 10^7 \text{ dynes per gm. per cm.}$$

The ratio of the force per unit mass developed electromagnetically in copper to the stress that can be carried by a centimeter length of steel having a unit mass is

$$\frac{2.61 \times 10^4}{8.96 \times 10^7} = 2.91 \times 10^{-4}.$$

This figure indicates qualitatively that a device that develops forces mechanically can be made much lighter, i.e., with much less mass, than one that develops forces electromagnetically. Consequently it appears desirable to design the motor so as to introduce the minimum possible inertia and to amplify by mechanical means the mechanical power thus obtained.

Although the force per unit mass produced electromagnetically is roughly a constant, i.e., depends relatively little on the size of the motor, the torque per unit of moment of inertia is inversely proportional to the radius. Thus, considering a unit mass of copper at a radius r, the torque developed is

$$2.61 \times 10^4 r \text{ dyne cm. per gm.}$$

while the moment of inertia is

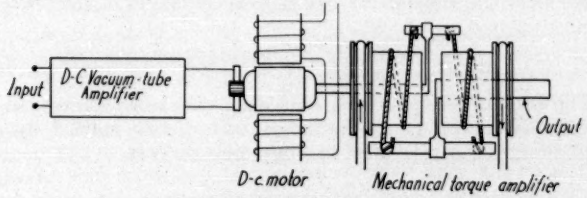
$$r^2 \text{ gm. cm.}^2 \text{ per gm.}$$

To obtain a high torque-inertia ratio, then, it is necessary to use a small armature radius.

In the above reasoning, no account has been taken of the inertia of armature iron, since the armature iron can be made stationary and its inertia thereby eliminated.

From these considerations it is evident that for a high-performance servo it is desirable to use a small electric motor followed by mechanical torque amplification. The limit in the reduction of the motor radius is fixed by considerations that will appear as the design progresses. From the above, however, it is seen that the power level at which a change from electrical to mechanical form is made should be low.

FIG. 2.



Schematic diagram showing principal apparatus elements of the servo-mechanism design.

Summarizing the general features of the proposed design, the servo-mechanism consists of a d-c. vacuum-tube amplifier, and a small d-c. motor driven from this amplifier. The motor in turn drives the input of a mechanical torque amplifier whose output is the output of the servo-mechanism. These elements are shown schematically in Fig. 2, which also shows the action of the torque amplifier.

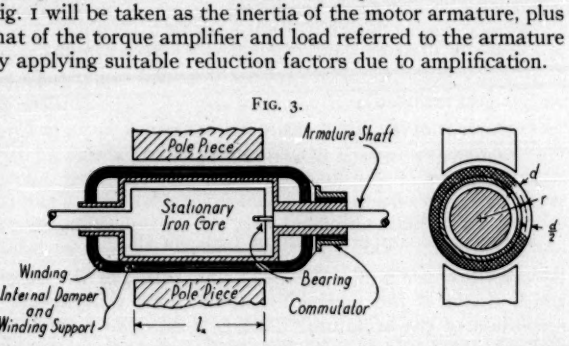
This completes the general layout of the device from a qualitative point of view and attention will be turned to the third aspect, viz., the quantitative design.

QUANTITATIVE DESIGN.

In this section, the problem is to determine suitable proportions and dimensions for the motor and torque amplifier so that the best performance shall be obtained, taking into account any practical considerations affecting construction. Performance is measured in terms of the ratio of maximum torque to moment of inertia of the output element shown in Fig. 1, subject to the restriction that the servo response must be aperiodic. The general procedure in the design is to formulate an expression for this ratio in terms of design variables and to determine what values of these variables make the ratio a maximum.

The dynamic system to be considered is that of Fig. 1, in which the output element is taken to be the motor armature, since the parameters of the torque amplifier and load can be readily referred to it. That is, the output-element inertia of Fig. 1 will be taken as the inertia of the motor armature, plus that of the torque amplifier and load referred to the armature by applying suitable reduction factors due to amplification.

FIG. 3.



Armature construction shown schematically. The iron armature core is made stationary to reduce inertia.

In Fig. 3 the type of armature construction used is shown. The armature is overhung about the stationary iron core, the armature shaft having one bearing as shown and another at the other end of the armature shaft.

In the formulation of the ratio of maximum torque to inertia the following quantities are used, numerical values being given in the units indicated:

r = mean radius of active conductor position (cm.),

l_a = axial length of active conductor (cm.),

d = radial depth of winding space (cm.),

J_a = moment of inertia of entire armature (gm. $\overline{\text{cm.}}^2$),

J_1 = moment of inertia of shaft load of motor referred to motor (gm. $\overline{\text{cm.}}^2$),

$J = J_a + J_1$ = total effective moment of inertia at motor shaft (gm. $\overline{\text{cm.}}^2$),

τ_a = torque developed by armature (dyne cm.),

τ_1 = steady-state load torque taken by load referred to motor shaft (load torque assumed to be Coulomb friction) (dyne cm.),

β = flux density in air gap (gausses),

q = armature copper current density due to impressed current (amperes per $\overline{\text{cm.}}^2$),

$$s = \frac{\text{volume of active copper}}{\text{volume of gross space occupied by active winding}}$$

= space factor for active copper (i.e., end connections are not included),

$$m = \frac{\text{moment of inertia of entire armature}}{\text{moment of inertia of active copper}},$$

$$\psi = \frac{\text{pole arc}}{\text{pole pitch}},$$

δ = density of copper conductor (gm. per $\overline{\text{cm.}}^3$).

By active copper is meant all of that portion of the copper which traverses the magnetic field at some time during a revolution of the armature. Although aluminum would be preferable from the point of view of performance, copper was chosen for this design because of the practical difficulty of soldering and handling very small aluminum wire.

It should be pointed out that q as defined above is not necessarily the current density resulting from the actual armature current. It is only the current density due to that

component of armature current that is impressed on the motor by an outside source. In general another component of current is set up in the armature by its own rotational or counter e.m.f. This component, discussed below, is not considered here as it has to do with damping rather than restoring forces.

Both the maximum torque and the moment of inertia can be expressed in terms of the above quantities. The dimensions that maximize their ratio then give the desired design.

In terms of these quantities

$$\tau_a = \frac{2\pi r^2 d\psi l_a \beta s q}{10}, \tag{3}$$

$$J_a = 2\pi r^3 d s m l_a \delta. \tag{4}$$

The desired ratio is

$$\frac{\tau_a - \tau_1}{J_a + J_1}. \tag{5}$$

Hence it is evident that the characteristics of the motor load must be fixed before proceeding further. This consists of a torque amplifier driving a prescribed load, which is taken as one pound-foot of Coulomb friction (constant torque). A complete study would include the investigation of the torque amplifier design. In this instance, however, it was found by preliminary calculation than an available torque amplifier with an over-all amplification of about 2000 and a moment of inertia, referred to its input shaft, of about 20 gm. $\overline{\text{cm}}.^2$, including a reasonable allowance for the inertia of its load, gave a reasonable design. A higher amplification would yield a motor too small for practical construction. As the inertia of the torque amplifier plus load was somewhat less than that of the motor itself, it appeared that the available torque amplifier design was approximately the optimum one for this purpose. These facts become evident as the design proceeds. With the above torque amplifier the assumed load torque of one pound foot, when referred through the torque amplifier amplification, becomes approximately 7000 dyne cm. (5) then becomes

$$\frac{\tau_a - 7000}{J_a + 20},$$

which is the expression to be maximized. When τ_a is given its maximum value it is evident from (2) that

$$4\gamma^2 M = \frac{\tau_a - 7000}{J_a + 20}$$

or using (3) and (4) that

$$4\gamma^2 M = \frac{0.2\pi r^2 d\psi l_a \beta s q - 7000}{2\pi r^3 d s m l_a \delta + 20}. \tag{6}$$

Before attempting to maximize (6) certain simplifications are desirable. Several of the factors have rather definite practical limits which when inserted simplify the expression. Thus

$$\left.\begin{array}{ll} \psi = 0.6 & \text{(very nearly constant),} \\ \beta = 7500 & \text{gausses,} \\ s = 0.25 & \text{estimated space factor,} \\ m = 4 & \text{by estimate,} \\ \delta = 8.9 & \text{for copper.} \end{array}\right\} \tag{7}$$

Although the values for s and m are only approximate, they are suitable for a preliminary design, in the light of which more accurate values can be obtained if desired.

Before inserting the values (7) in (6) the constants including these values will be lumped thus:

$$c_1 = 4 \times 2\pi\delta m d s l_a,$$
$$c_2 = 0.2\pi\psi\beta d s l_a.$$

With these values (6) becomes

$$\gamma^2 M = \frac{c_2 q r^2 - 7000}{c_1 r^3 + 80}, \tag{8}$$

which can be written

$$r^3 - \frac{c_2 q}{\gamma^2 M c_1} r^2 + \frac{80}{c_1} + \frac{7000}{\gamma^2 M c_1} = 0. \tag{8a}$$

The problem is to determine the value of r which makes M in (8a) a maximum. By trial and error it is found that practical proportions are obtained with $d = 0.2$ cm., $l_a = 3.0$ cm., and $q = 600$ amperes $\overline{\text{cm.}}^2$ (about 500 circular mils per ampere) and for these values the maximum of M is 1160 with $r = 1.1$

cm. This is for $\gamma^2 = 1$ or critical damping. An examination of (8a) shows that other values of γ^2 lead to the same best value of r for which $\gamma^2 M = 1160$. In other words, making $\gamma^2 < 1$ does not alter the best proportions but it does increase the value of M obtained. It is also found that altering M from 4 to 3 alters the best proportions but little, the best value of r becoming 1.2 cm. with a corresponding $\gamma^2 M$ of about 1400. Furthermore small changes in r from the best value change the value of $\gamma^2 M$ but little. Hence while some of these factors cannot be fixed with accuracy in the preliminary design, the proportions for best performance can be determined with sufficient accuracy by using approximate values.

Thus far the value of q has been merely assumed without justification other than that the order of magnitude is reasonable assuming that the component of current due to counter e.m.f. is small relative to the impressed current. By putting the numerical values already found for the other variables into (6), the effect of varying q can be ascertained. Thus with

$$c_1 = 8\pi 8.9 \times 4 \times 0.2 \times 0.25 \times 3.0 = 134.2,$$
$$c_2 = 0.2\pi 0.6 \times 7500 \times 0.2 \times 0.25 \times 3.0 = 424,$$
$$r = 1.1,$$

the expression for $\gamma^2 M$ is

$$\gamma^2 M = 1.98q - 27.1$$
$$= 1190 - 27.$$

Evidently for all values of q of the same order as that assumed and for the value of r found, $\gamma^2 M$ can be assumed proportional to q. In (8a), from which the optimum value of r and $\gamma^2 M$ are simultaneously obtained, the proportionality between q and $\gamma^2 M$ eliminates the effect of q in the second term. As the fourth term is less than 8 per cent of the third term, the effect of a change in $\gamma^2 M$ is small in the sum of these two terms. Consequently q can be varied over a considerable range without significantly altering the optimum value of r. This is very convenient, since any value of q, as determined subsequently by armature heating and by the necessity for adequate damping, so long as it is not less than say one-tenth of the assumed value, can be used in (9) to obtain $\gamma^2 M$.

Increasing l_a or d in the above design would improve $\gamma^2 M$ slightly but would introduce practical difficulties. Thus a longer armature would be considerably more difficult to wind and the larger overhang from the bearing would be undesirable mechanically. A significant increase in d would result in a poor design magnetically. Consequently it appears that the dimensions thus far obtained represent approximately the optimum design consistent with good construction.

Although the optimum physical dimensions have been obtained, the maximum attainable value of $\gamma^2 M$ has not yet been determined, since it depends upon q which has not been fixed. The current density q due to impressed current is limited by two considerations. The first is armature heating; the second is the available damping which, with a given γ^2, limits the amount of restoring torque that can be utilized. The exact way in which these two limitations actually occur is not sufficiently simple to be evident without analysis. The heating limitation will be considered first.

Evidently the total heat that can safely be generated in the armature mass per unit time is limited by the ability of the armature to dissipate heat. Knowing the dimensions of the armature, its approximate heat dissipating ability is readily calculated. The next problem, then, is to relate q to this allowable armature heating.

There are two sources of heat in the armature, first the armature winding itself, and second, the internal eddy-current damper. The total heat to be dissipated by the armature is that generated in both of these sources. Because of the low heat-storage capacity of the light armature and its consequent liability to rapid overheating due to transient overloads, the permissible steady-state heating limit is set at the conservatively low figure of 0.04 watt per square centimeter or approximately 0.25 watt per square inch.

In analyzing the heating the following quantities are involved in addition to those already defined:

q_0 = actual current density in armature copper at maximum steady-state angular velocity ω_m (amperes per $\overline{\text{cm.}}^2$),

q = current density in armature copper due to restoring-torque component of armature current at angular velocity ω_m (same quantity as previously defined),

q_a = current density in armature copper due to damping-torque component of armature current at angular velocity ω_m, i.e., component of armature current set up by armature back e.m.f.,

ρ = volume resistivity of copper (ohms per $\overline{\text{cm.}}^3$),

V = volume of copper in armature ($\overline{\text{cm.}}^3$),

V_a = volume of active* copper in armature ($\overline{\text{cm.}}^3$),

$c = \dfrac{V}{V_a}$,

f_a = armature torque per unit angular velocity due to armature back e.m.f. (dyne cm. per radian per sec.),

f_d = armature torque per unit angular velocity due to internal eddy-current damper,

H_a = heat generated in armature copper at angular velocity ω_m (ergs per sec.),

H_d = heat generated in internal eddy-current damper at angular velocity ω_m,

H = total armature heating at angular velocity ω_m.
$= H_a + H_d$,

A_s = heat dissipating area of armature ($\overline{\text{cm.}}^2$).

It is readily seen that the total armature heating is

$$\begin{aligned} H &= H_a + H_d \\ &= 10^7 q_0^2 \rho c V_a + f_d \omega_m^2 \end{aligned} \tag{9}$$

and that this heating is limited to the value

$$H \lesssim 0.04 A_s. \tag{10}$$

Several of the variables can be evaluated numerically for the present design. Thus

$$\begin{aligned} V_a &= 2\pi r l_a d s \\ &= 2\pi 1.1 \times 3.0 \times 0.2 \times 0.25 \\ &= 1.04 \ \overline{\text{cm.}}^3 \end{aligned}$$

Assuming that the average end connection has a length equal to l_a

$$c = 2.$$

* See page 552 for definition of "active copper."

The armature area for dissipating heat may be taken as a cylinder capped by two hemispheres. Then

$$A_a = 2\pi\left(r+\frac{d}{2}\right)l_a + 4\pi\left(r+\frac{d}{2}\right)^2$$
$$= 2\pi 1.2 \times 3 + 4\pi 1.2^2$$
$$= 41 \text{ cm.}^2$$

and

$$H \leqslant 0.04 \times 41 = 1.6 \text{ watts}$$
or 1.6×10^7 ergs per second.

With

$$\rho = 1.73 \times 10^{-6} \text{ ohms per cm.}^3$$

and the other foregoing numerical values in (9) there results

$$H = 10^7 \times 1.73 \times 10^{-6} \times 2 \times 1.04 q_0^2 + f_d \omega_m^2,$$

which in (10) gives as the heating criterion

$$36 q_0^2 + f_d \omega_m^2 \leqslant 1.6 \times 10^7 \text{ ergs per sec.} \qquad (11)$$

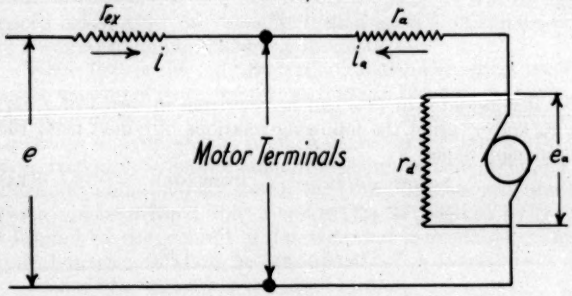
Equation (11) gives the heating limitation on the resultant current density q_0 but not explicitly that on q. However, by a consideration of the circuit to which the motor armature is connected, a relation between q_0 and q can be obtained, which put in (11) will give the desired heating limitation on q. This relation, which is applicable to steady-state conditions, will now be developed.

The current density q_0 is the resultant of two component current densities. The first component q is due to the component i of armature current set up by an external source of voltage. The second component q_a is due to the component i_a of armature current set up by the counter e.m.f. developed in the armature itself. Heating in the armature copper in general depends upon the numerical difference of these two components because in the steady state they flow in opposite directions.

If the circuit supplying the motor is the equivalent of an independently adjustable voltage of e volts in series with a resistance of r_{ex} ohms external to the motor, the equivalent circuit for the power source and motor is that shown in Fig. 4. The resistance r_d represents the effect of the eddy-current

damper, for, assuming that the inductive reactance of the eddy-current paths is negligible compared to their resistance, the damping torque due to these currents is proportional to the armature counter e.m.f. e_a. It should be noted in Fig. 4,

FIG. 4.



Equivalent circuit of motor armature, eddy-current damper, and power source.

that the actual armature current is that in r_a and not that in the source of e_a.

Since

$$q - q_a = q_0$$

and

$$\frac{q}{q_a} = \frac{e}{e_a},$$

$$q = \frac{q_0}{1 - \frac{e_a}{e}}. \qquad (12)$$

From the foregoing discussion it is seen that the component current density q which results in restoring torque in the motor, can be greater than the resultant or actual current density q_0, provided the source has other than an infinite impedance, so that e_a can set up a component of armature current. In this case the current component i_a results in a damping torque.

In (12) the e's can be replaced by mechanical parameters. Thus, since the resultant steady-state armature torque and

therefore the armature current is a maximum when the motor is operating at the maximum angular velocity ω_m and lag angle θ_m, and since, for this condition, the damping torque $f_a\omega_m$ due to e_a, and the restoring torque $k\theta_m$ due to e are proportional respectively to these voltages, (12) can be written

$$q = \frac{q_0}{1 - \frac{f_a\omega_m}{k\theta_m}}. \tag{13}$$

The denominator of (13) can be rewritten in terms of J, M, f_d, θ_m and γ^2 using the following relations obtained from the companion paper

$$\omega_m = \sqrt{M\theta_m} \qquad \text{from (95)}, \tag{14}$$

$$k\theta_m = 4\gamma^2 JM \qquad \text{“ (96)}, \tag{15}$$

$$f = 2\gamma\sqrt{Jk} \qquad \text{“ (92)},$$

$$= 4\gamma^2 J\sqrt{\frac{M}{\theta_m}}. \tag{16}$$

From (16)

$$f_a = f - f_d = 4\gamma^2 J\sqrt{\frac{M}{\theta_m}} - f_d. \tag{17}$$

Substituting (14), (15) and (17) in (13), there results

$$q = q_0\frac{4\gamma^2 J}{f_d}\sqrt{\frac{M}{\theta_m}}. \tag{18}$$

The quantity M can be eliminated from (18) by using (8) written as

$$\gamma^2 M = a_1 q + a_2, \tag{19}$$

in which

$$a_1 = \frac{c_2 r^2}{c_1 r^3 + 80}$$

and a^2 was shown by (9) to be negligible. (19) in (18) gives

$$q_0^2 = q\frac{f_d^2\theta_m}{16\gamma^2 J^2 a_1}. \tag{20}$$

Using (14), (19) and (20) in the limiting case of (11) gives q_h, the maximum permissible value of q as limited by heating.

$$q_h = \frac{1.6 \times 10^7 \gamma^2}{f_d \theta_m \left(\frac{2.25 f_d}{J^2 a_1} + a_1 \right)}. \tag{21}$$

Equation (21) is one of the two necessary limitations upon q and is thus one of the two results sought. It will be discussed below after the other limitation upon q, that imposed by required damping, has been obtained.

While (21) is the only limitation, due to the motor itself, on the maximum restoring torque that can be used, the proper operation of the servo as a whole imposes another limitation. To secure a given γ^2, i.e., a given degree of oscillation in the servo response, a definite amount of damping is required, i.e., a definite value of f as shown by (16). In this way the q which can be utilized, and therefore the M as given by (19), is limited by the amount of damping that is available. This limitation on q will now be considered.

From (19), neglecting the small term a_2

$$q = \frac{\gamma^2 M}{a_1}. \tag{22}$$

From (2)

$$M = \frac{\tau_m}{4\gamma^2 J} = \frac{k\theta_m}{4\gamma^2 J}. \tag{23}$$

From (16)

$$k = \frac{f^2}{4\gamma^2 J}. \tag{24}$$

Using (24) in (23) and the result in (22) gives the value q_d of q as limited by the available damping. Thus

$$q_d = \frac{f^2 \theta_m}{16\gamma^2 J^2 a_1}$$

$$= \frac{\theta_m}{16\gamma^2 J^2 a_1} (f_a + f_d)^2. \tag{25}$$

Equations (25) and (21) fix the definite limits upon the value of q that can be used. If

$$q > q_h$$

the armature will overheat. If

$$q > q_d$$

the servo response will be more oscillatory than that prescribed by the given γ^2.

The performance possibilities of this servo can be readily visualized by putting in the numerical values already obtained for the parameters in (25) and (21) and plotting the results. The value of J is given by

$$\begin{aligned} J &= J_a + J_1 \\ &= 2\pi r^3 dsml_a\delta + 20 \\ &= 2\pi 1.1^3 \times 0.2 \times 0.25 \times 4 \times 3.0 \times 8.9 + 20 \\ &= 65 \text{ gm. } \overline{\text{cm.}}^2. \end{aligned}$$

Also

$$a_1 = \frac{\gamma^2 M + a_2}{q} = \frac{1160 + 27}{600} = 1.98.$$

Using these values in (21) and (25) the limits for q in this motor become

$$q_h = \frac{5.95 \times 10^{10}}{\frac{\theta_m f_d}{\gamma^2}(f_d + 7360)} \tag{26}$$

and

$$q_d = 7.47 \times 10^{-6} \frac{\theta_m}{\gamma^2}(f_a + f_d)^2. \tag{27}$$

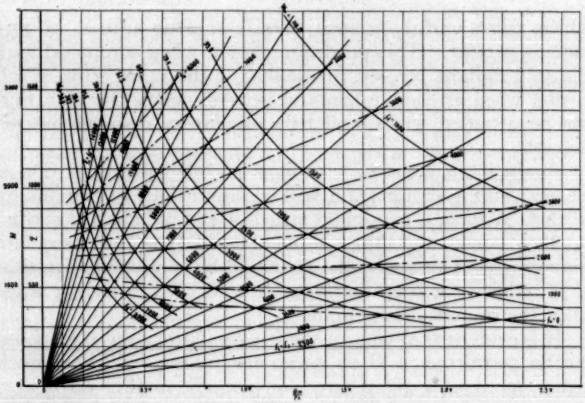
In Fig. 5, q_h and q_d are plotted from (26) and (27) as functions of the variable θ_m/γ^2 with f_d and f_a as parameters. Since q_h and q_d are limiting values of q, the scale of ordinates is given in terms of q only. By taking θ_m/γ^2 as the independent variable rather than θ_m, Fig. 5 is applicable for all values of γ^2. Curves of constant f_a were drawn in by connecting points of constant difference between f and f_d.

It may be mentioned that the response of this servo to a suddenly-applied constant angular velocity for each of several values of γ^2 is shown in Fig. 12 of the companion paper.

The design possibilities of this servo are effectively brought out by the curves of Fig. 5. Suppose for example that it is

desired to operate this servo with a maximum error of $\pi/2$ radians or one-quarter of a revolution, and that the coefficient f_a of damping due to the armature winding is limited in a manner discussed below to 4000 dyne cm. per radian per second. Aperiodic response is required, i.e., $\gamma^2 = 1$. How much eddy-current damping should be used to get the maximum possible speed and what is this maximum speed? Entering the abscissa scale of Fig. 5 at 0.5π and reading off the value of maximum velocity corresponding to $f_a = 4000$, it is found that

FIG. 5.



Performance possibilities of servo-motor prior to fixing tolerated error and armature counter e.m.f. damping.

$\omega_m = 49.2$ radians per second approximately, and that the necessary f_d is 4230 dyne cm. per radian per second. It is also found that the restoring torque component of current should be such as to work the given amount of copper in the armature at 782 amperes per $\overline{\text{cm}}.^2$ at this displacement angle of 0.5π. This is equivalent to fixing the restoring torque coefficient k, which can be readily determined from q by (22) and (23).

If on the other hand an error of π radians or one-half

revolution can be tolerated, it is seen that a maximum speed of 75.0 radians per second can be obtained and that f_d should be 2120 dyne cm. per radian per second.

It is interesting to compare the response obtained above with that obtainable with the same limits of error but with somewhat less than aperiodic damping. Thus consider a γ^2 of 0.477 which gives a response as represented by curve 3 of Fig. 12 in the companion paper. Retaining the permissible error of 0.5π radians, the abscissa with which to enter Fig. 5 is

$$\frac{\theta_m}{\gamma^2} = \frac{0.5\pi}{0.477} = 1.048\pi,$$

which taken with $f_a = 4000$ fixes f_d at 2100 and ω_m at 77.2 radians per sec. This maximum speed, when compared with the 49.2 radians per sec. attainable at the same error of following when aperiodic damping is used, shows the gain that can be attained when a slightly oscillatory response is permissible.

Again if an error of π radians or one-half revolution is allowed with a γ^2 of 0.477 it will be found that, corresponding to an f_a of 4000, a maximum speed of 125 radians per second can be attained, and that f_d should be 910 dyne cm. per radian per second. This speed of 125 should be compared with that of 75 radians per second attainable with the same error angle when the response is aperiodic.

It is evident that when a slightly oscillatory servo response can be permitted, a given servo-mechanism will give much higher speed at a given error, or a much smaller error at a given speed than it will when adjusted for critical damping. It should be noted that a γ^2 of 0.477 corresponds to an overshoot of only about 8 per cent. in the output velocity of a servo when it is called upon to follow a suddenly-applied constant velocity. However, in the present application, aperiodic response is required, and the gains just pointed out, which may be utilized in other applications where a slightly oscillatory response can be tolerated, are not available here.

Thus far no definite performance requirements have been laid down, except that the figure of merit for the servo is to be a maximum and that the response is to be non-oscillatory. Consequently the entire range of design represented in Fig. 5

with $\gamma^2 = 1$ is available from which the design best suited to the application in hand will be selected.

This servo-mechanism is to drive a recording pencil having a total travel of 25 inches and the maximum permissible error of following is set at 0.1 per cent. or 0.025 inch. In addition the servo is to drive a light shutter and possibly other apparatus. Preliminary estimate has shown that if the total output range is 500 revolutions, the maximum torque required from the servo output will be safely within the capacity of the torque amplifier. Consequently 500 revolutions of the servo output is taken as full range and the maximum allowable error is thereby fixed at 0.5 revolution or π radians.

It is of interest to digress momentarily at this point to mention that with a torque amplifier having a higher amplification and output torque, which can drive the load through gears stepping up the speed, the figure of merit can in general be improved. This statement while not immediately obvious can be justified by consideration of Fig. 5 and of the effect of speed changes by gears on the figure of merit. Because of the problems involved in obtaining a torque amplifier with higher amplification for this servo, the limitations set by the available torque amplifier were accepted as definite.

Attention is now returned to the design. With the error fixed at π radians, it is evident from Fig. 5 that if f_a, f_d or their sum is fixed, the design is fixed. It happens in this case that f_a, the coefficient of armature counter e.m.f. damping, is the limited factor as will now be seen.

A convenient means of expressing f_a, which depends upon the design thus far made and the total armature-circuit resistance, is in terms of the ratio of armature resistance to total armature-circuit resistance. Thus by considering all of the armature copper as making a single-turn winding, short-circuited through an external resistance r_{ex} such that

$$\frac{r_a}{r_a + r_{ex}} = b_1$$

it can be seen that,

$$f_a = \frac{2\pi r^3 \beta^2 \psi^2 ds}{10^9 \rho} \frac{2l_a^2}{(mlt)} b_1, \qquad (28)$$

where

(mlt) = mean length of turn of armature winding.

For the armature as designed

$$f_a = \frac{2\pi 1.1^3 \times 7500^2 \times 0.6^2 \times 0.2 \times 0.25}{10^9 \times 1.73 \times 10^{-6}} \times \frac{2 \times 3.0^2}{13} b_1$$
$$= 6780 b_1.$$

Since the armature current is to be supplied by a vacuum-tube amplifier, it will probably be desirable to use very small wire in the armature winding both in order to use the amplifier efficiently and to secure high armature damping. Trial armature winding and amplifier designs bear out this conclusion, and indicate that No. 43 B and S copper fits well with an amplifier of suitable power output. Such a winding will have a number of conductors

$$\frac{2\pi rds}{\text{area No. 43 } B \text{ and } S} = \frac{2\pi 1.1 \times 0.2 \times 0.25}{2.46 \times 10^{-5}}$$
$$= 14{,}030.$$

and a resistance at 20° C. of

$$\frac{14{,}030}{4}\left(\frac{mlt}{2}\right)(R \text{ per cm.}) = \frac{14{,}030 \times 13 \times 0.0696}{4 \times 2}$$
$$= 1590 \text{ ohms.}$$

The amplifier subsequently described has an effective resistance to an increment of voltage applied at its output terminals of approximately 4200 ohms. With this resistance, the coefficient f_a of counter e.m.f. damping becomes

$$f_a = 6780 \frac{1590}{1590 + 4200}$$
$$= 1860 \text{ dyne cm. per radian per sec.}$$

In Fig. 5 this value of f_a taken with the prescribed error of π radians shows that approximately 3100 dyne cm. per radian per sec. of eddy-current damping should be used giving a value for f of 1860 + 3100 or 4960 cyne cm. per radian per sec., and that the armature maximum current density q due to the restoring torque component of current should be about 580

amperes per $\overline{\text{cm.}}^2$ Since the armature winding has two paths in parallel, the maximum terminal armature current with the winding of No. 43 B and S wire becomes

$$580 \times 2 \times 2.46 \times 10^{-5} = 0.0286 \text{ ampere}$$

which is approximately what the tentatively designed amplifier will deliver when working into a resistance of 1600 ohms. The armature design is now complete and only the damper design remains to be considered.

Since the damper also serves as the mechanical support for the armature winding (see Fig. 3), it must have adequate mechanical strength as well as the desired electrical characteristics. The choice of material is therefore left open until the thickness of the cylindrical damper shell is determined as a function of the electrical resistivity of the damper material. A rigorous design of the damper would call for a plot of current distribution in the shell from which the effective resistance could be determined. Since the ends of the cylinder are made relatively heavy in order to secure mechanical strength, and the current in the sides of the cylinder flows principally in the area directly under the poles, the assumption that the resistance is all in the area under the poles appears to be a fair approximation. Actually it proved to be a very close approximation. On this basis it can be seen that the damping coefficient f_d, computed as the torque for a unit angular velocity of the armature, is

$$f_d = \frac{2\pi\left(r - \frac{d}{2}\right)^3 \beta^2 \psi l_a t}{10^9 \rho},$$

where

t = thickness of damper shell in cm.

ρ = resistivity of shell material in ohms per $\overline{\text{cm.}}^3$

For this motor

$$f_d = \frac{2\pi(1.1 - 0.1)^3 7500^2 \times 0.6 \times 3.0}{10^9} \frac{t}{\rho}$$

$$= 0.636 \frac{t}{\rho}.$$

It is found that a copper damper giving the required f_d of 3100 is only 0.0084 cm. thick which is too thin for adequate strength. With brass the thickness t is

$$t = \frac{3100 \times 8.2 \times 10^{-6}}{0.636} = 0.040 \text{ cm. or } 0.0157 \text{ inch,}$$

which is satisfactory mechanically. While the brass damper is heavier than one of copper or aluminum, the additional weight is necessary for mechanical strength.

It is of interest at this point to determine the time scale for Fig. 12 of the companion paper which shows the nature of the transient response of this type of servo. The time constant T in the companion paper is

$$T = \frac{2J}{f}$$

$$= \frac{2 \times 65}{(1860 + 3100)} = 0.0262 \text{ second.}$$

This is a small time constant for such apparatus and indicates very fast following. The response of this servo to a suddenly-applied input angular velocity of ω_1 radians per second is shown in Fig. 12 of the companion paper by using the above value of T in the coördinates. Thus each unit of abscissa is 0.0262 seconds. It is seen that the transient practically disappears within less than 4×0.026 or 0.1 second for any γ^2 while for $\gamma^2 = 0.289$ the final steady-state operating angle is reached in $0.75 \times 0.0262 = 0.0196$ second, following which a certain amount of overshoot occurs before the steady-state is reached.

This completes the basic design in accordance with which the servo was built. A few details of the physical construction may be of interest, however. The armature winding actually used consists of nine coils of 800 turns each of No. 43 *B* and *S* enamelled copper wire, laid over a layer of insulation on the smooth cylindrical surface of the eddy-current damper and held in place with a silk thread binding. The nine-segment commutator, made from 0.030-inch silver sheet, is approximately 3/8 inch in diameter and receives current from two carbon brushes. Since the length of the air gap in

the magnetic circuit is comparable with that of much larger d.-c. machines, the field coils have to be rather large in comparison with the armature to produce the relatively high air-gap density desired. Figure 6 shows the assembled armature and pole pieces. In Fig. 7 the motor is shown coupled to the torque amplifier into which it is designed to work.

FIG. 6.

Photograph of armature and pole pieces. The armature has been partly withdrawn from the field and the brush holders removed. The outside diameter of the armature is approximately one inch.

Having thus completed the design, attention will be turned to the tests made on the servo as actually constructed.

TESTS OF SERVO-MECHANISM.

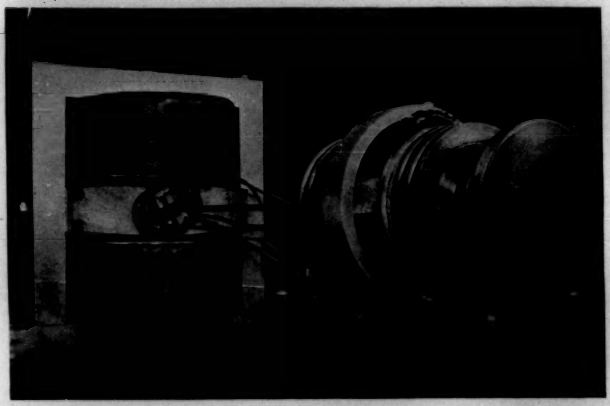
In this section the methods of testing the servo-mechanism, including the amplifier supplying the motor armature, the motor itself, the torque amplifier, and the necessary coupling circuits will be described briefly, and the results of the tests given together with computed checks.

The tests are divided into two groups, first those by which the various constants necessary for the computation of the performance are determined, and second, those by which the

overall transient and steady-state performance of the servo-mechanism is determined.

Measurement of Constants.—As seen from the foregoing analysis, the essential constants are three: first, the moment of inertia, J, of the entire mechanism referred to the motor shaft; second, the damping constant f; and third, the resilience constant k. From these three constants the entire transient

FIG. 7.



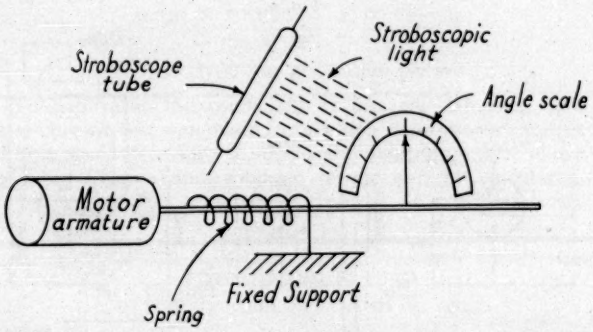
View of the motor and torque amplifier. The motor armature, shown in Fig. 6, is obscured by the brush holder. The field coils and field structure show clearly, however. Since the air gap and air-gap magnetic flux density are of the same order as those found in much larger machines, the field coils are large relative to this armature.

and steady-state performance can be predicted. Of these the first can be determined by a simple test of the parts whose inertia is desired. The second and third constants, f and k, however, are functions of the associated apparatus as well as of the motor. The determination of J will be considered first; then, following a brief description of the complete servo assembly as set up for performance tests, the measurement of f and k will be described.

Because of the difficulty sometimes encountered in readily measuring the moment of inertia of small light apparatus, the

technique of applying a standard method will be briefly described. Fig. 8 shows schematically the experimental set-up. The frequency of free oscillation of the system consisting of the armature and shaft inertia, and the resilience of a spiral spring was determined by adjusting the frequency of the stroboscopic light [3] used to illuminate the pointer until the two frequencies coincided. Although the amplitude of oscillation decreased to one-half its initial value in about one second, independent observers measured the frequency of 13.72 cycles per second with a consistency of a fraction of one per cent. A measurement of the spring constant in terms of weights and

FIG. 8.



Schematic diagram of set-up for the experimental determination of the moment of inertia of the motor armature.

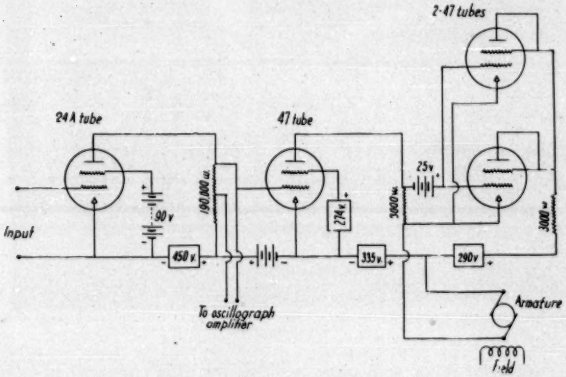
a measured lever arm gave a value for this constant of 2.56 $\times 10^5$ cyne-cm. per radian. From these two measurements the value of the moment of inertia, J, of the armature and shaft, is computed to be 34.4 gm-$\overline{\text{cm}}$.^{2} To this must be added the inertias of the torque amplifier and of the load, referred to the motor shaft. These inertias were computed for the test set-up and referred to the motor shaft by dividing each actual value by the measured torque amplification between the motor shaft and the member under consideration. In this way the additional inertia was found to be 21.4 gm. $\overline{\text{cm}}$.^{2} making a total of 55.8 gm. $\overline{\text{cm}}$.^{2} effective at the motor arma-

ture. It will be noted that the test figures for the armature and total inertia are somewhat below those used in the design, hence the performance should be somewhat better than that predicted with the design data.

Using the spring constant as a standard of torque, the motor armature-torque constant was found to be 1.12×10^7 dyne cm. per ampere at an arbitrary standard field current.

Since, as mentioned above, one component f_a of the damping coefficient f depends upon the source of power for the armature, the amplifier from which the motor was driven during the tests will be briefly described before discussing the measurement of f. Figure 9 shows the essential elements of

FIG. 9.



Circuit of d.-c. amplifier used in servo performance tests.

this amplifier, consisting of two stages with simple resistance coupling. The last stage has two sets of tubes working in opposition as a means of avoiding the low current or power efficiency of a bucking-battery scheme for reversing the armature current. By a proper adjustment of biases, this circuit gives a very nearly linear current response with respect to input grid voltage for a range of -20 to $+20$ milliamperes running current in the motor armature which has an apparent

running resistance of approximately 5500 ohms. The departure from linearity at ± 30 milliamperes of motor armature current is only 10 or 15 per cent. An increment of approximately 0.115 volt applied to the grid of the first tube changes the running armature current of the motor from zero to 20 milliamperes.*

The damping coefficient f_a due to back e.m.f. in the motor armature can be found from the fact that with a given grid voltage on the first tube, the difference between the armature current when the motor is stalled and when it is running at 700 r.p.m. is 10 milliamperes. From the torque per ampere given above, the coefficient f_a is found to be

$$f_a = \frac{0.010 \times 1.12 \times 10^7 \times 60}{700 \times 2\pi}$$

$$= 1530 \text{ dyne cm. per radian per sec.}\dagger$$

To determine the coefficient of internal damping f_d, a speed-current test was made from which the armature torque per unit speed necessary to supply the damping torque plus a negligible friction and windage torque can be calculated. At 700 r.p.m. the armature current is 20.4 milliamperes. Then

$$f_d = \frac{20.4 \times 1.12 \times 10^7 \times 60}{700 \times 2\pi}$$

$$= 3120 \text{ dyne cm. per radian per sec.,}$$

which is very close to the design figure. The coefficient of total damping f is

$$f = f_a + f_d = 4650 \text{ dyne cm. per radian per sec.}$$

Measurement of Transient and Steady-State Response.—Having determined the necessary constants for calculating the transient and steady-state response of the servo-mechan-

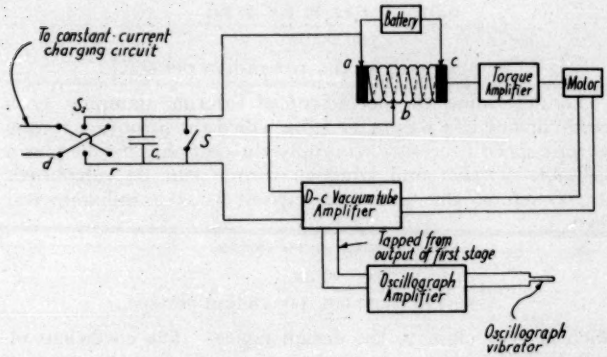
* The amplifier as described has since been much improved by eliminating all batteries, by reducing to about one-half the maximum potential difference previously appearing within the amplifier, by so adjusting the circuits that the amplifier is insensitive to supply line fluctuations, and by somewhat increasing the current capacity of the output.

† This figure is somewhat below that of 1860 used in the design due to changes made in the amplifier in the interim between design and test of the motor.

ism, the apparatus shown schematically in Fig. 10 was set up in order to measure this response.

The essential feature of this set-up is the "back-coupling" from the servo-motor output to the input of the amplifier that supplies power to the motor armature. Whenever the motor armature is not at the equilibrium position this "back-coupling" causes an armature current to flow, which sets up a torque tending to move the armature toward the equilibrium position. In Fig. 10 it is seen that the angular position of the armature is measured by the voltage appearing between

FIG. 10.

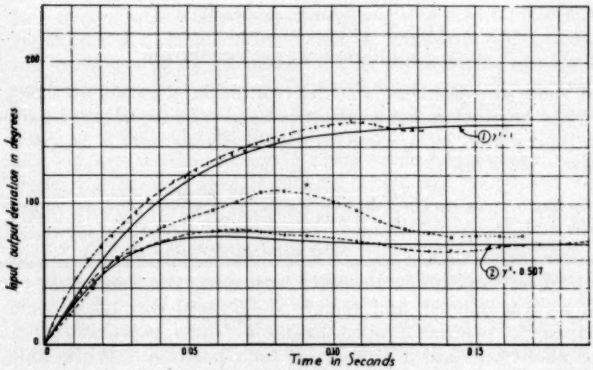


Schematic diagram of set-up used for test of transient and steady-state performance of servo-mechanism.

points *a* and *b* of a spirally-wound drop wire which is rotated by the armature under the sliding contact *b*. This voltage is fed into the amplifier input, thus coupling the output to the input. With this coupling, the servo-mechanism would tend to follow any shaft on which another drop wire similar to *ac* was placed, if the voltage from it were connected in series with *ab* and the amplifier input. Instead of making the servo follow the motion of such a shaft for this test, it was made to follow a voltage building up at a constant rate simulating the voltage obtainable from the drop wire on a hypo-

thetical input shaft when this shaft receives a suddenly-applied constant angular velocity. This voltage was obtained as shown in Fig. 11 from a 50.2 microfarad condenser, C_1, charged in either direction by the plate current of a Type 24A vacuum tube adjusted to give constant plate current. By adjusting the charging current, any desired input speed could be simulated. By means of the oscillograph amplifier, consisting of four 71A tubes in parallel, the magnitude of the amplifier input signal, which is a direct measure of the follow-

FIG. 11.



Curves of input-output deviation or error angle as a function of time when a constant angular velocity is suddenly impressed on servo input.
———— Calculated responses.
x x x x x Test points $\gamma^2 = 1$ or critical damping.
○ ○ ○ ○ ○ Test points $\gamma^2 = 0.51$ less than critical damping. Maximum deviation exceeded linear range of amplifier.
+ + + + + Test points. Same as circles except that amplifier linear range was not exceeded.

ing error angle θ, was recorded oscillographically. With this set-up it is only necessary to adjust k to the proper value by adjusting the amplifier gain or the voltage applied to the drop wire, and to adjust the condenser charging current to correspond to the desired impressed input velocity, and the servo-mechanism is ready for test.

Two tests were made, the first with k set for critical damping or $\gamma^2 = 1$; the second with k set for less than critical

damping, viz., with a γ^2 of approximately 0.5. In the first, the critically damped case,

$$k = \frac{f^2}{4J} = \frac{4650^2}{4 \times 55.8} = 97{,}000 \text{ dyne cm. per radian.}$$

This value of k is easily realized by calculating the armature current which should flow for some arbitrary angular armature displacement, and adjusting the amplifier gain or slide-wire voltage until this displacement from equilibrium causes the calculated current to flow. In this case the current per unit angle is

$$\frac{97{,}000}{11.2 \times 10^6} \text{ amp. per radian}$$
$$= 54.3 \text{ milliamperes per revolution.}$$

The slide-wire gradient for this condition was set at 0.205 volt per revolution, hence an impressed velocity of 500 r.p.m. was represented by a condenser charging current of (capacitance = 50.2 mfd.)

$$i = C\frac{dE}{dt} = \frac{50.2}{10^6} \times \frac{0.205 \times 500}{60}$$
$$= 85.8 \text{ microamperes.}$$

With these conditions properly adjusted, the switch S_1 of Fig. 10, was opened, and switch S_2 operated through a cycle consisting of position d, open, position r, open, position d, etc., with about one-half a second for each position. While this cycle was in progress an oscillogram was taken of the output of the oscillograph amplifier together with a 60-cycle timing wave. By calibrating the oscillograph amplifier output in terms of the error angle θ, the oscillograph record is readily converted to a curve of error angle *vs.* time. In Fig. 11, curve A is the calculated response for this case. The test points are taken from the oscillogram for the transient following the change of switch S_2 from the open position to position d. The ordinates are the values of error angle in degrees plotted as a function of time in seconds for a suddenly-impressed angular velocity of 500 r.p.m. The agreement is good considering that the system contains a d.-c. amplifier having a voltage gain factor of 1000, a torque amplifier having a torque

gain factor of 1000, and that the resilience and slight backlash within the torque amplifier were neglected in the analysis.

In the second test k was adjusted to 192,000 dyne cm. per radian, giving a γ^2 of 0.507. With a slide-wire voltage gradient of 0.412 volt per revolution, a condenser charging current of 170 microamperes corresponds to 491 r.p.m. In Fig. 11, curve B shows the calculated response in terms of the error angle θ. The circled points are from an oscillogram taken with the motor starting from rest and show much more overshoot than the calculated curve. This discrepancy is due to the fact that the amplifier is closely linear in response only for currents corresponding to angles up to about 65° or 70°, above which the gain decreases somewhat. Once the angle goes above this value during a transient, the overshoot will be larger than calculated. The cross points are for the stopping transient, during all of which the amplifier is operating in the linear range. In this case the actual error of following is in reality the steady-state error of 71° less the ordinates plotted, but as the change in error angle for this case is precisely the same function as the previously calculated error angle, the experimental points are plotted on the actual error scale for comparison with the calculated curve. It will be noted that the agreement between the experimental and calculated curves is remarkably good for the case in which the assumption of a constant k, i.e., a linear amplifier, is in accord with the facts. The slightly greater oscillation in the experimental curve may likely be due to overshoot within the torque amplifier itself, since an angular displacement of 5° to 10° may occur between its input and output shafts under heavy load conditions such as occur during large angular accelerations.

Operation was tried with a γ^2 of 0.25 but the torque amplifier tended to develop sustained oscillations with this small amount of damping, so no recorded tests were made for this condition.

It will be noted that the design was conservative and that the test performance was slightly better than that predicted by the design calculations. For example, instead of a time constant T of 0.0262 second as computed from design data, the tests on the apparatus as constructed showed a value of 0.0239 second. This is due to the actual moment of in-

ertia being slightly smaller than that on which the design was based. However, the check between the performance as computed from the test values of the parameters J, k, and f, and the experimentally determined performance is very good.

IMPROVEMENT OF PERFORMANCE.

It is of interest to consider briefly the ways in which the servo design given above could be improved by extending certain of the design limitations. The figure of merit is proportional to the maximum torque that can be employed and inversely proportional to the inertia. Improvement results from an increase of the former or a decrease of the latter. These two possibilities will be considered in order.

The maximum torque is limited primarily by two factors, first, the permissible heating of the armature, and second, the amount of damping available. These two factors are interrelated. One way of improving performance is to allow higher dissipation in the armature. As the limit set in this design is conservative, experience with the actual operation of this servo would undoubtedly show that more heating can be tolerated. In Fig. 5 the ordinates of the f_d = constant curves are directly proportional to the allowable heat dissipation per unit of armature area. It is likely that a factor of two or more could be obtained in the figure of merit by this means.

Another way of improving the figure of merit by increasing the maximum torque is to employ a source of armature power having a lower internal resistance. In this way the counter e.m.f. component of damping is increased. Increasing the coefficient f_a in this way has the dual effect of reducing the net or heat-producing armature current and increasing the available damping. Both effects contribute to a higher figure of merit. Since for a given motor, f_a is proportional to the ratio of armature resistance to the combined resistance of armature and power source, it is evident that beyond a certain point, the reduction of the source resistance has little effect. In the present design where r_a = 1800 ohms (measured) and the source resistance r_{ex} = 4200 ohms, it can be seen that the source resistance would have to be reduced by more than a factor of three to double f_a. Figure 5 shows that it is not possible to obtain any major improvement by this method.

The second method of improving the servo performance is to reduce the inertia. Lighter construction can be used but here again the possible gain is rather small. By careful design the torque amplifier could be lightened and the optimum size of the motor thereby reduced. Some gain can be realized by this method but at the cost of ruggedness and ease of construction.

By employing all of these methods an increase of a factor of five in the figure of merit could probably be obtained.

Much more promising is the method, discussed in the companion paper, whereby the restoring torque is made a linear function of the first two time derivatives of the input-output deviation as well as of the deviation itself. The possibility of improvement by this method is theoretically without limit. Practical considerations of stability fix a very real limit in any particular case. Nevertheless the possibilities are undoubtedly great, and where exceptionally high speeds of response and great accuracy are desired this method is the most promising.

CONCLUSIONS.

In this paper a high-speed response servo-mechanism has been designed in accordance with criteria established in the companion paper. The servo consists of a d.-c. vacuum-tube amplifier feeding a small, very low-inertia d.-c. motor which in turn drives the output through a mechanical torque amplifier. In the design the optimum dimensions for the motor are determined, and the nature of the various limitations on performance is analyzed in considerable detail. In particular the limitations due to armature heating and available damping are presented in the form of curves from which the best apportionment of damping for prescribed conditions of servo operation can be determined by inspection.

The tests, by which the parameters and the overall performance of a servo constructed on the above design were measured, are described and the results given. The experimental curves of response giving error angle as a function of time check closely with calculations except for minor discrepancies due probably to factors not considered in the analysis.

The response of this servo is very fast, the transient following the sudden application of a constant input velocity lasting only about 0.1 second when critical damping is employed. With slightly less than critical damping, the steady-state deviation angle is reached in about 0.04 second, followed by a slight overshoot which is damped out within a cycle.

Various possibilities for improving the performance are considered briefly, the most significant possibility being the use of the velocity and acceleration of the input-output deviation, as well as the deviation itself, to control the restoring torque.

ACKNOWLEDGMENTS.

The writer is indebted to Dr. V. Bush for his ever helpful interest and suggestions, and to Messrs. G. S. Brown, D. B. Sinclair, and S. E. Caldwell for their help in the design, construction and test of the apparatus.

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DISTRIBUTION OF THE NONPROTEIN NITROGEN IN THE ALASKA PEA.

BY

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The pea (*Pisum sativum*) is one of our most valuable vegetables. The very fact that the pea gave rise to a big industry speaks for itself. As to the economic significance of this legume it may suffice here to mention that from 1925 to 1930 about 200,000 tons of peas were produced for canning purposes annually, and from 1927 to 1930, 5 million to 7 million bushels were grown for marketing in the fresh state.[1] This means that the value of the annual pea crop amounts to 20 to 25 millions of dollars. All of which goes to show the importance of a thorough knowledge regarding this vegetable.

Its frequent use in our diet and its general popularity are due chiefly to its palatableness and flavor. The qualities mentioned are especially marked in young unripe peas whose superiority consists partly in their higher sugar and lower starch content,[2] as compared with ripe peas which have a lower sugar and higher starch content. Furthermore, the pea is one of the vegetables that is richest in nitrogen and protein content.[2]

So far as the writer is aware, the chemical work on peas in this country is ordinarily confined to the estimation of the sugars, starch, crude fiber, ash, total and protein nitrogen.[3] While knowledge of those compounds is useful and even necessary it practically gives chiefly information on the energy-producing constituents (carbohydrates).

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[1] U. S. Dept. Agr. Yearbook 1933, p. 543.

[2] Jodidi and Boswell, Jour. Agr. Research, **48**, 703 (1934).

[3] Boswell, *Proc. Amer. Soc. Hort. Science*, **21**, 178 (1924); **23**, 162 (1926); Sayre, Willaman, and Kertesz, Geneva Agr. Exp. Sta., Tech. Bul. 176 (1931); Horsfall, Kertesz, and Green, *J. Agr. Research*, **44**, 833 (1932); Kertesz and Green, Ibid., **45**, 361 (1932).

It is known from the work of Osborne and Campbell [4] that the pea contains several proteins, namely the globulins legumin and vicilin, the albumin legumelin, and a proteose. But it is also true that the publications mentioned fail to give information on the *nonprotein* nitrogen compounds which in young peas make up more than 50 per cent of the total nitrogen. The nonprotein nitrogen in plants is made up of amino acids, acid amides, polypeptides, organic bases, alkaloids, lecithins, phosphatides, glucosides, etc. It is obviously important to know which of those compounds occur in the pea and in what proportions. Preliminary work [5] on the nonprotein nitrogen as obtained with the Alaska pea grown in 1929 may have given results which are not necessarily valid for peas raised in 1931. It may be emphasized here, too, that the primary object of the 1929 work was to find out the chemical nature of the *humin* nitrogen. Moreover, the results of 1929 were secured with peas grown on two experimental plots, one not fertilized and the other fertilized with muriate of potash. Inasmuch as the nonprotein nitrogen of peas from these two plots showed certain differences in composition, it seemed probable that peas grown in 1931 on variously treated plots would show still other differences. Although the hydrolysis of the 1929 peas was effected with dilute sulfuric acid, it was decided to hydrolyze the 1931 peas with dilute hydrochloric acid whose hydrolytic effect is generally known to be stronger than that of dilute sulfuric acid. For these reasons the distribution of the nonprotein nitrogen in the 1931 peas might reasonably be expected to differ in several respects from that of the 1929 peas, giving at the same time a broader and safer foundation on which to base our conclusions.

METHODS EMPLOYED.

The total nitrogen was estimated according to the Gunning modification of the Kjeldahl method. The protein nitrogen was determined according to the Stutzer method.[6] The nitrogen distribution was made according to Hausmann's method,[7]

[4] Osborne and Campbell, *Jour. Amer. Chem. Soc.*, **20**, 358 (1898).
[5] Jodidi, *Jour. Agr. Research*, **43**, 811 (1931).
[6] Stutzer, *Jour. Landw.*, **28**, 103 (1880).
[7] Hausmann, *Zeit. Physiol. Chem.*, **27**, 95 (1899); **29**, 136 (1900).

as modified by Osborne and Harris,[8] and as employed by Jodidi and by Jodidi and his associates in previous investigations,[9] as well as by Boswell.[10] The nitrogen of polypeptides was hydrolyzed by means of 20 per cent. hydrochloric acid in accordance with the observations of Fischer,[11] and the amino nitrogen of the resulting hydrolysate was estimated by the Van Slyke method.[12] Other methods used are described later.

EXPERIMENTAL PART.

The peas of grade 4 [13] used in these experiments were grown in 1931 at the Arlington Experiment Farm, Rosslyn, Va., on three plots. One plot was fertilized with nitrate of soda and sulfate of ammonia at the rate of 400 and 280 pounds per acre, respectively; another plot received 300 pounds of muriate of potash per acre; the third plot received no fertilizer. In order to prevent decomposition through the action of enzymes or microorganisms, the peas soon after being harvested were dried at a temperature of 60° to 65° C. in a strong draft of air. For preliminary orientation the protein nitrogen in the peas was estimated. It was found that on a fresh-weight basis the peas from the nitrogen-treated and potash-treated plots and from the untreated plot contained 1.131, 1.127, and 1.131 per cent. of total nitrogen, respectively. These amounts consisted of 68.02, 69.72, and 69.29 per cent. of protein nitrogen, respectively, leaving 31.98, 30.28, and 30.71 per cent. of nonprotein nitrogen, as calculated on the basis of the total nitrogen. It will be seen that peas from the nitrogen-treated plot contained a somewhat larger proportion of total nitrogen in the nonprotein form than did those from the other plots.

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[8] Osborne and Harris, *Jour. Amer. Chem. Soc.*, **25**, 323 (1903).

[9] Jodidi, *Jour. Agr. Research*, **43**, 811 (1931); Jodidi and Peklo, Ibid., **38**, 69 (1929); Jodidi, Moulton, and Markley, *Jour. Amer. Chem. Soc.*, **42**, 1061, (1920); **42**, 1883 (1920).

[10] Boswell, Maryland Agr. Expt. Sta. Bul. 306 (1929).

[11] Fischer, Untersuchungen über Aminosäuren, Polypeptide und Proteine, Berlin, p. 23, 1906.

[12] Van Slyke, *Jour. Biol. Chem.*, **9**, 185 (1911); **12**, 275 (1912); **16**, 131 (1913–14).

[13] Peas of grade 4 will not pass through a screen having round holes eleven thirty-seconds of an inch in diameter, but will pass through holes twelve thirty-seconds of an inch in diameter.

In order to obtain an insight into the chemical nature of the nonprotein nitrogen, the following experiments were made with the peas from each of the various plots: Each of six flasks, containing 25 g. of pea meal, was treated with 500 c.c. of hot ammonia-free water and kept on the water bath for half an hour, when the contents was centrifuged, as filtration was practically impossible because of the large quantity of starch present. The solid residues in the centrifuge tubes were again treated in like manner. All extracts thus obtained were concentrated on the steam bath and centrifuged for the second time. The supernatant liquid of the centrifuge tubes was transferred to a 500-c.c. flask and made up with water to the 500-c.c. mark. In two portions of 25 c.c. each the nitrogen was estimated according to the Kjeldahl method, to ascertain the total nitrogen content of the aqueous extract. In order to remove the protein, the remaining 450 c.c. of the extract was boiled and 1 c.c. of acetic acid and a small quantity of tannic acid was then added, and the whole was allowed to stand for about 24 hours. At the expiration of this time the extract was centrifuged and the supernatant liquids of the centrifuge tubes were collected and made up with water to 500 c.c. Two 10-c.c. portions of this solution were Kjeldahlized, and the nitrogen content of the protein-free extract was thus determined.

ESTIMATION OF AMIDE NITROGEN.

The remaining 480 c.c. of the protein-free solution was divided into two equal portions. Each of these, after the addition of 24 c.c. of concentrated hydrochloric acid, was hydrolyzed in a 1-liter Kjeldahl flask under a reflux condenser, by boiling for 2 hours in a glycerin bath. On cooling, the contents of each flask was almost neutralized with sodium hydroxide solution and subjected to distillation with magnesium oxide previously reduced to cream, the evolved ammonia being absorbed in receiving flasks containing $N/10$ sulphuric acid. The results obtained are recorded in table 1.

HUMIN NITROGEN OBTAINED ON HYDROLYSIS OF THE ACID AMIDES.

After the ammonia resulting from hydrolysis of the acid amides had been distilled off, each of the two residues remain-

TABLE I.

Nitrogen of Acid Amides in the Alaska Pea, Expressed as Percentage of the Total Nonprotein Nitrogen in the Protein-free Extract from Peas Grown in Soil Treated with Nitrogen and with Potash and in Untreated Soil.

Treatment of Soil.	Distillation No.	Total Nonprotein Nitrogen. Mg.	Amide Nitrogen.	
			Mg.	On Basis of Total Nonprotein Nitrogen. Per Cent.
Fertilized with nitrogen...	1	565.728	50.310	8.89
	2	565.728	51.711	9.14
	Average	565.728	51.011	9.02
Fertilized with potash.....	3	531.002	57.401	10.81
	4	531.002	58.251	10.97
	Average	531.002	57.826	10.89
Unfertilized.....	5	498.124	41.045	8.24
	6	498.124	39.950	8.02
	Average	498.124	40.498	8.13

ing in the Kjeldahl flasks was treated with 25 c.c. of very hot ammonia-free water, brought to a boil, and allowed to settle. The supernatant liquid was then decanted on a Büchner funnel provided with a hardened filter. This treatment was repeated about 10 times to make certain that all soluble nitrogen was removed. Finally each of the residues was transferred to its respective Büchner funnel and washed well with hot ammonia-free water, all filtrates and washings being reserved for the estimation of the amino and peptide nitrogen. In order to obtain the black substance quantitatively, the following procedure was adopted: Suction under the Büchner funnels was allowed to go on until the humin substance became dry, when it was transferred mechanically from the funnels to 800-c.c. Kjeldahl flasks as completely as possible, traces being rinsed down by the wash bottle. The last traces were dissolved in dilute sulfuric acid and added to the Kjeldahl flasks. The nitrogen determinations accroding to the Kjeldahl method yielded, respectively, 31.7046 and 33.6380 mg. of nitrogen for the extract from the nitrogen-treated soil, 35.3647 and 37.0639 mg. for the extract from the potash-treated soil, and 45.3292 and 46.3753 mg. for the extract from the unfertilized soil. These figures correspond, respectively, to 5.60 and 5.95 (average 5.78) per cent., 6.66 and 6.98 (average 6.82) per cent., and 9.10 and 9.31 (average 9.22) per cent. of humin

nitrogen, when calculated on the basis of the total nonprotein nitrogen.

ESTIMATION OF AMINO NITROGEN.

All filtrates and washings from the two residues, the oxidation of which had yielded the humin nitrogen, were together concentrated on the water bath to small volume. The result was a dark-brown sticky sirup, permeated with a considerable quantity of salt, which was readily soluble in water, even at room temperature, but practically insoluble in strong alcohol. The sirup on cooling was treated with 96 per cent. alcohol, which, with the water in the sirup made approximately an 85 per cent. alcohol. The whole was, on mixing, filtered by suction on a Büchner funnel provided with two soft filters, and washed repeatedly with 85 per cent. alcohol. This was followed by extraction of the salt with alcohol of the same strength, in a Soxhlet apparatus, until the siphoning alcohol appeared practically colorless. The salt thus obtained from peas grown in soil treated with nitrogen and with potash and in untreated soil weighed 37, 34, and 40 g., respectively, in the alcohol-moist state. It was identified as sodium chloride mixed with a very small quantity of magnesium salt. All filtrates, washings, and extracts from the salt were united and further concentrated to small volume. The insignificant quantity of salt in the resultant sirup was separated as already outlined. The final filtrates and washings were on further concentration made up to 100 c.c. This was divided into two equal portions. One 50-c.c. portion was used directly for the estimation of amino nitrogen according to Van Slyke's nitrous acid method; the other 50-c.c. was applied to the determination of peptide nitrogen. The results of the amino nitrogen estimation recalculated to 50 c.c. of the solution are given in Table 2.

HUMIN NITROGEN OBTAINED ON HYDROLYSIS OF THE POLYPEPTIDES.

To the 50 c.c. of the solution reserved for the estimation of the peptide nitrogen a sufficient quantity of concentrated hydrochloric acid was added to make a concentration of 20 per cent., and the whole was boiled for 12 hours in an 800-c.c. Kjeldahl flask under a reflux condenser, in a glycerin bath at about 125° C. The hydrolysate, containing a quantity of

TABLE 2.

Nitrogen of Amino Acids in the Alaska Pea, Expressed as Percentage of the Total Nitrogen in Extract of Humin Substances from Peas Grown in Soil Treated with Nitrogen and with Potash and in Untreated Soil.

Treatment of Soil.	Analysis No.	Total Nonprotein Nitrogen in 50 c.c. of Solution. Mg.	Amino Nitrogen in 50 c.c. of Solution.	
			Mg.	On Basis of Nonprotein Nitrogen. Per Cent.
Fertilized with nitrogen	1	393.61	37.6350	9.67
	2	385.07	37.9900	9.76
	Average	389.34	37.8125	9.72
Fertilized with potash	3	344.02	26.9024	7.74
	4	350.90	28.1071	8.09
	Average	347.46	27.5048	7.92
Unfertilized	5	355.72	30.8054	8.75
	6	348.14	29.6267	8.42
	Average	351.93	30.2161	8.59

black substance, was then evaporated to dryness on the water bath in order to remove the hydrochloric acid, and the residue was taken up with hot water. The humin substance present was filtered out and washed well with hot ammonia-free water, filtrate and washings being reserved for the determination of peptide nitrogen. The black substance remaining on the filter was quantitatively transferred to the flask in which hydrolysis had taken place, and the whole was oxidized according to the Kjeldahl method. On distillation the oxidized humin substance yielded, in the form of ammonia, 109.3621, 66.0521, and 58.6315 mg. of humin nitrogen for the peas grown in soil treated with nitrogen and with potash and in untreated soil, respectively. These figures correspond, respectively, to 28.09, 19.01, and 16.66 per cent. of humin nitrogen, when calculated on the basis of the nonprotein nitrogen.

ESTIMATION OF PEPTIDE NITROGEN.

The filtrate and washings from the humin substance obtained after hydrolysis with 20 per cent. hydrochloric acid were quantitatively transferred to a porcelain dish and concentrated on the water bath to very small volume. The residue was then filtered through a folded filter, washed, and made up with water to 50 c.c. Two 10-c.c. portions of this solution

were oxidized according to Kjeldahl's method, to ascertain the total nitrogen present. In three other portions of 10 c.c., 10 c.c., and 5 c.c., respectively, the amino nitrogen was determined by Van Slyke's method. The results recalculated to 10 c.c. of the solution are presented in Table 3.

TABLE 3.

Peptide Nitrogen in the Alaska Pea, Expressed as Percentage of Total Nitrogen in Extract of Humin Substances from Peas Grown in Soil Treated with Nitrogen and with Potash and in Untreated Soil.

Treatment of Soil.	Analysis No.	Total Nonprotein Nitrogen in 10 c.c. of Solution. Mg.	Peptide Nitrogen Plus Amino Nitrogen (Originally Present in Solution) in 10 c.c.		Peptide Nitrogen Only, on Basis of Nonprotein Nitrogen. Per Cent.
			Mg.	On Basis of Nonprotein Nitrogen. Per Cent.	
Fertilized with nitrogen.....	1	45.8968	19.5360	42.68	32.96
	2	45.6586	19.9791	43.64	33.92
	3	—	19.5420	42.69	32.97
	Average	45.7777	19.6857	43.00	33.28
Fertilized with potash.....	4	42.5032	18.6042	43.94	36.02
	5	42.1770	18.5301	43.76	35.84
	6	—	18.7293	44.24	36.32
	Average	42.3401	18.6212	43.98	36.06
Unfertilized.....	7	43.9204	20.7402	47.11	38.52
	8	44.1272	20.9234	47.53	38.94
	9	—	21.2947	48.37	39.78
	Average	44.0238	20.9861	47.67	39.08

For the sake of convenience the results on the 1931 peas and the data on the 1929 crop are presented in Table 4. The latter contains only the averages expressed in percentages of the total nonprotein nitrogen. A glance at the table shows the differences in the nonprotein nitrogen of the various pea samples.

It is not out of the question that the nitrogen distribution in peas of smaller sizes might differ somewhat from that of grade 4 as given in this paper.

SUMMARY.

A study was made of the distribution of nitrogen in the Alaska pea, grade 4, grown during 1929 and during 1931 in

TABLE 4.

Distribution of the Nonprotein Nitrogen in the Alaska Pea Grown in Soil Treated with Nitrogen and with Potash and in Untreated Soil, in 1929 and 1931.

Year	Treatment of Soil.	Nitrogen of Acid Amides. Per Cent.	Humin Nitrogen Obtained on Hydrolysis of Acid Amides. Per Cent.	Nitrogen of Amino Acids. Per Cent.	Humin Nitrogen Obtained on Hydrolysis of Polypeptides. Per Cent.	Peptide Nitrogen. Per Cent.	Residual Nitrogen. Per Cent.
1929	Not treated	7.58	[a]10.51	8.43	[c]11.72	41.83	19.93
1929	300 lbs. of muriate of potash per acre	11.43	[a]6.73	6.04	[c]21.47	37.64	16.69
1931	Not treated	8.13	[b]9.22	8.59	[c]16.66	39.08	18.32
1931	300 lbs. of muriate of potash per acre	10.89	[b]6.82	7.92	[c]19.01	36.06	19.30
1931	400 lbs. of sodium nitrate and 280 lbs. of ammonium sulfate per acre	9.02	[b]5.78	9.72	[c]28.09	33.28	14.11

[a] The humin nitrogen was obtained on hydrolysis of the purified pea extract with a 5 per cent. sulfuric acid solution for 2 hours.
[b] The humin nitrogen was obtained on hydrolysis of the purified pea extract with a 2.1 per cent. hydrochloric acid solution for 2 hours.
[c] The humin nitrogen was obtained on hydrolysis of the peptide nitrogen with a 20 per cent. hydrochloric acid solution for 12 hours.

untreated soil and in soil treated with nitrogen and with potash. The percentage of amide nitrogen was found to be lowest in peas grown in untreated soil both in 1929 (7.58) and 1931 (8.13) and highest in peas grown in potash-treated soil both in 1929 (11.43) and 1931 (10.89), whereas in peas grown in nitrogen-treated soil it was between the two (9.02). On the other hand, the percentage of amino nitrogen was lowest in peas grown in potash-treated soil (6.04) in 1929 and 7.92 in 1931) and highest in nitrogen-treated soil (9.72), while the percentage of amino nitrogen in peas grown in untreated soil was between the two (8.43 in 1929 and 8.59 in 1931). The percentage of peptide nitrogen was lowest in peas from nitrogen-treated soil (33.28), higher in peas from potash-treated soil (36.06 in 1931 and 37.64 in 1929), and highest in peas from the untreated soil (39.08 in 1931 and 41.83 in 1929).

There appears to be fairly good agreement between the seasons 1929 and 1931 so far as the distribution of the nonprotein nitrogen in the peas is concerned. For instance, the percentage of amide nitrogen in peas grown in potash-treated soil was 11.43 in 1929 and 10.89 in 1931, and the percentage of amino nitrogen in peas grown in untreated soil was 8.43 in 1929 and 8.59 in 1931.

Since the amino acids and proteins are known to have high nutritive value, it is probable that the polypeptides, which in the nitrogen metabolism stand between the amino acids on the one hand and the proteins on the other, are also nutritious. In summing up the more or less nutritive constituents, namely, the amide, amino, and peptide nitrogen, it will be found that the percentage of nutritive nonprotein nitrogen in the peas investigated is as follows: 57.84 and 55.11 in peas of 1929 grown in untreated soil and in soil treated with potash, respectively; 55.80, 54.87, and 52.02 in peas of 1931 grown in untreated soil and in soil treated with potash and with nitrogen, respectively. These figures show plainly that the nutritive values of peas grown in 1929 and 1931 in soils treated alike are very similar. The actual percentages of nutritive nonprotein nitrogen in the peas from the variously treated plots are somewhat higher, being modified by the presence of humin nitrogen. This is due to the fact, as shown in a pre-

vious publication,[5] that the latter contains in part peptide linkings which on hydrolysis yield free amino acids known to be nutritious. On the other hand, it should be borne in mind that the nonprotein nitrogen in peas of grade 4 makes up only about 30 to 32 per cent of the total nitrogen.

[5] Jodidi, *Jour. Agr. Research*, 43, 811 (1931).

Female Plant Secretions.—It seems to be only a matter of time before science workers will have discovered many interesting similarities between the life processes of plants and animals. Probably the present outstanding example is the close resemblance of the life-supporting plant product, chlorophyll, to the corresponding constituent in animal blood, the hemoglobin. The chemical constitution and function of these two products are surprisingly alike. Chemically, the important difference is that magnesium constitutes the inorganic element of chlorophyll while iron forms the keystone in hemoglobin.

Just recently, B. SKARZYNSKI reported the results of a chemical examination of the female flowers of the willow tree. Through proper treatment he was able to extract a small quantity of folliculin hydrate from ten thousand times its weight of flowers. Folliculin hydrate has been known to chemists for some time and usually has been obtained from well-known animal sources. It is classified as a sex hormone and may be obtained in appreciable quantities from the urine of pregnant female vertebrates. Folliculin hydrate produces estrogenic action and mice injected with it exhibit sexual excitement.

C.

Bibulous Weeds.—(U. S. D. A. Clip Sheet No. 845.) Ordinarily the farmer can afford to tolerate such weeds as may grow on the uncultivated premises of his farm. Under certain conditions, however, such growth becomes quite costly. According to O. V. STOUT of the Bureau of Agricultural Engineering, weeds and water-loving plants in irrigated regions of the West use more water in proportion to the ground they occupy than do the common run of crops. Farmers in the arid region pay for water to irrigate crops and unless they eradicate water-absorbing weeds they also pay for water to grow them.

Cattails and bulrushes will consume, acre for acre, four times as much water as average crops. An acre of heavy stand of smartweed may consume in a season's growth enough water for the irrigation of three acres of alfalfa. Kelp, dock, western goldenrod, prickly lettuce, cocklebur and nettle may use twice as much water as alfalfa.

C.

ON THE AIR FLOW PAST A HEATED SPHERE.

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1. THE PROBLEM.

The experiments described in this paper relate to the flow caused by the motion of a solid in a fluid otherwise at rest. Ever since modern science came into existence this problem, on account of its particular importance in navigation and in other fields, has aroused the curiosity of the foremost minds. With the advent of aeronautics and the intensive study of aerodynamics the efforts to solve this problem have been greatly multiplied. The goal of the many theoretical investigations and experimental researches undertaken along this field has as yet not been reached. There exists to-day no satisfactory and rational explanation of what goes on in the fluid through which a solid moves, sufficient to enable us to predict the type of fluid motion in any particular instance, nor to compute the fluid forces associated therewith.

The present investigation falls in line with the many others which were undertaken to solve this problem. In common with them all, this paper constitutes but a step likely to advance us closer to the solution, rather than containing the solution itself.

2. THEORETICAL CONSIDERATIONS.

The most useful concept for the study of fluid motion along solid boundaries is the "boundary layer" introduced by L. Prandtl (1).[1] Prandtl was the first to appreciate the need for a particular classification of the regions of a moving fluid with respect to the type of its motion. It was well known before him that the fluid behind a moving solid is left in a state of more or less turbulent agitation, as opposed to the quiet and almost imperceptible motion near its front and

[1] Numbers in parentheses refer to bibliography at the end of paper.

along its sides. Prandtl realized that this large region of apparently quiet motion should again be subdivided into two parts. In the first region, located at some distance from the boundary walls, the flow is practically unaffected by viscosity (potential flow). In the second region, directly adjacent to the walls, the viscosity forces as well as the inertia forces are of importance in determining the type of the flow.

This stratum adjacent to the surface has been called by Prandtl "boundary layer" (Grenzschicht). Making some plausible simplifying assumptions, he proceeded to investigate mathematically the flow inside the boundary layer. As far as it went the theory proved a success. Prandtl and his associates were able to explain rationally and in numerical detail some experimental facts unexplained before.

Unfortunately, the final outcome of these discoveries and investigations fell short of expectations. It developed that the application of Prandtl's differential equations was limited to a class of fluid motions which only rarely occur in technical problems. The equations are limited to steady laminar flows. Prandtl assumes the stability of this flow without further examination. Experience has shown however that in more technical problems the steady laminar motion is unstable and is replaced by a turbulent motion. This type of motion is stable but unsteady, just as the waving of a flag in stable equilibrium is unsteady. Both flag and flow perform an unsteady, more or less periodic motion, fluttering and oscillating, and this type of motion is persisting and takes the place of the simpler laminar flow within the boundary layer suggested by Prandtl.

A rational solution of the flow problem requires therefore a full understanding of the details of motion in an unsteady and turbulent boundary layer. This term has been retained from the beautiful researches of Prandtl, and it has been extended to signify the thin layer of fluid adjacent to the solid boundary, even when, in that region, the flow deviates to a much larger degree from a potential flow than was suggested by Prandtl. This turbulent layer has been mathematically investigated using assumptions based on experimental data, and some progress can be expected by pursuing this line of thought. However, no strictly rational treatment of the

fluid motion in a turbulent boundary layer is possible as long as we are unacquainted with the reasons for the instability of the laminar boundary layer, and with the details of the turbulent motion taking the place of the laminar motion in the layer.

The present paper is devoted to the determination of the regions which govern the type of flow. Since the solid by its presence causes the motion, the regions very far from it cannot be expected to have much influence on the flow. Neither may the regions directly adjacent to the solid boundary be governing, since the walls, by completely shearing off the normal velocity component, and by damping the tangential velocity component, force these layers to move in a definite manner. The more specific question suggests itself, whether it is the inner or the outer parts of the boundary layer which govern the flow, or perhaps different laminae at different velocities. The following experiments have been undertaken to obtain an answer to this question. The answer may prove a valuable clue to the solution of the whole problem.

3. A NEW APPROACH.

The method adopted for obtaining information on this question is novel. It is known that equal values of equivalent Reynolds numbers indicate dynamic similarity of all conditions precedent, and hence are the necessary condition for the dynamic similarity of all conditions consequent. In other words, all non-dimensional coefficients describing the flow or its physical characteristics are functions of Reynolds number only. It is therefore a universal custom to study coefficients as functions of Reynolds number and to plot them accordingly. All points so plotted lie on a definite curve or a family of definite curves. For a fluid homogeneous in its physical properties, particularly in its kinematic viscosity, the Reynolds number of the set-up determines the type of flow coming into existence, for this Reynolds number is representative of all Reynolds numbers of the different portions of the flow.

More light on the mechanics of moving fluids is now sought by making the fluid surrounding the moving object non-homogeneous. The tests are so arranged that the kinematic viscosity of the fluid in the boundary layer is different from

the kinematic viscosity without. This makes it possible to identify the flow-governing region by the Reynolds number characteristic of the resulting flow. The type of flow is observed by determining the drag coefficient of the solid in question. At first the relation between the Reynolds number and the drag coefficient in a homogeneous fluid is experimentally determined. One definite drag coefficient corresponding to each Reynolds number results. Next a drag measurement is made in a non-homogeneous fluid, and the drag coefficient computed. The Reynolds number corresponding to the same value of drag coefficient in the homogeneous fluid is then determined. From this Reynolds number, the flow velocity, and the model size, the kinematic viscosity is computed which corresponds to the homogeneous flow. The region or layer of the non-homogeneous fluid possessing this characteristic viscosity is finally ascertained, and this region is considered flow-governing or representative.

The solid used in the present experiments was a hollow brass sphere. By heating the sphere, the temperature of a thin layer of air surrounding it is raised, and its kinematic viscosity becomes greater than that of the colder air further removed from the sphere.

The thickness of this heated layer of air is of the same order of magnitude as the thickness of the boundary layer. This is true for turbulent as well as laminar flow, and follows from the close similarity of the mechanisms of heat diffusion and momentum diffusion. For laminar flow, the mechanism by which the transfer of momentum is brought about (in other words the cause of the frictional resistance responsible for the existence of the boundary layer) is the same as the mechanism by which the transfer of heat takes place. In both instances the exchange is brought about by molecular diffusion between adjacent laminae. Our knowledge concerning the mechanism controlling turbulent motion is at present still rather incomplete. However experiments of F. Elias (2) show that even for this motion the distribution of velocity and temperature are almost identical near the heated surface. This is the case, although very careful measurements recently made by A. Fage (3) indicate that when the general motion is eddying the conditions of flow even very near the solid surface are not

the same as exist in a streamline flow, in other words no part of the boundary layer is laminar.

4. DESCRIPTION OF APPARATUS AND EXPERIMENTAL PROCEDURE.

The experiments were performed in a wind tunnel at the Catholic University grounds. The tunnel is of the Venturi type, and has a square cross-section of nine square feet in area. The air stream is generated by a four-bladed propeller driven by a 10 H.P. electric motor. The maximum air speed obtainable was 60 M.P.H., and the minimum constant air speed was about 25 M.P.H. The speed was varied by changing the R.P.M. of the motor. A wall plate located approximately three feet in front of the sphere and connected to an alcohol manometer was used for measuring the air speed. The wall plate had been calibrated against a pitot tube substituted for the sphere, so that the manometer readings indicated the speed of air in the region just approaching the sphere.

A hollow brass sphere of 5.01 inch outside diameter and 0.026 inch thickness was used for the experiments. It was equipped with a 660 watt electrical resistance heating unit. The electrical power input was measured by aid of an ammeter and voltmeter. The power input and sphere temperature were varied by means of a resistance in series with the heating coil. To measure the temperature of the sphere four copper-constantan thermocouples were connected to as many points inside the spherical shell, and to a Leeds and Northrup potentiometer. The temperature was sensibly constant over the entire surface of the sphere even at high air speeds, and all four thermocouples gave the same readings. The surface of the sphere was polished to reduce radiation losses.

Figure 1 shows a cross-section of the sphere used in the tests.

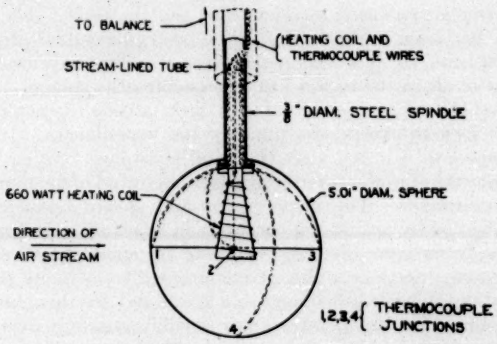
The drag of the sphere was measured by means of a balance of the moment type. The scale beam of the balance is graduated in thousandths of a pound, and readings can be estimated to a tenth of the smallest graduations. The sphere was attached to the balance through a steel spindle of three-eighth inch diameter, which had been hollowed over part of its length, so that the thermocouple and resistance wires entered the sphere without affecting the air flow around it. To further

prevent a disturbance of the flow a streamlined hollow tube was placed around the spindle up to a distance of two inches from the sphere.

During each test the room temperature and atmospheric pressure were observed.

It was originally intended to perform the tests in the so-called critical Reynolds number region, where the variation of the drag coefficient with Reynolds number is most pronounced. But the air speed which could be obtained in the tunnel was not sufficiently high to reach the critical Reynolds number for the set-up as it was first arranged.

FIG. 1.



Previous experiments have shown that one of the factors which determine the value of the critical Reynolds number for a flow around a sphere is the smoothness of the approaching air stream. Thus O. Flachsbart (4) found that by making the approaching flow turbulent by introducing into the tunnel a 10 × 10 mm. mesh wire screen, transversely to the air flow at 0.5 meter in front of the sphere, the critical Reynolds number of his set-up was reduced to about half its original value. In our experiments a similar method was adopted to reduce the critical Reynolds number.

In all, three series of experiments were performed. In the first there was no obstruction in the wind tunnel except a

honeycomb at the entrance which remained there throughout all tests, the purpose of which is to break up large scale variations in the entering air flow. A second set of tests was performed with a 3/8″ × 3/8″ mesh wire screen placed 20 inches in front of the sphere and covering the entire tunnel section (36″ × 36″). The introduction of the wire screen reduced the available air speed. In order to remedy this, and at the same time to increase the turbulence slightly, a third set of tests was made with a 3/8″ × 3/8″ mesh wire screen stretched only across the central (18″ × 18″) part of the tunnel section 15 inches in front of sphere. The results of the three series of tests are given in separate tables on pages 601, 603, 605 and the curves plotted from these results are shown in separate figures on pages 607, 608, 609.

For each change in the tunnel the wall plate was recalibrated against a pitot tube. Before the tests a spindle correction curve was obtained in the usual manner. The values of spindle drag thus obtained were then subtracted from the total drag, so that the resistance values used in computing the drag coefficient represent only the drag of the sphere.

Two different procedures were followed in performing the experiments. At first, the sphere temperature was kept constant, while the sphere resistance was measured at various air speeds. This was done with the sphere at room temperature, as well as at a few higher temperatures. Following this, the air speed was kept constant while the sphere temperature was changed and the corresponding drag measured. In all cases it was necessary to wait a little before taking readings, in order to make certain that sphere temperature and air speed had assumed constant values. The points plotted from the observations in these two different ways are designated by different symbols in the curves that follow. All points obtained by the second method are shown by solid dots.

Measurements were also made to determine the heat losses from the sphere at various air speeds and several values of temperature difference between sphere and air. This required very little change in the set-up. Information on the subject is lacking and the results may have some direct bearing on the problem.

5. EXPERIMENTAL RESULTS.

For the purpose of clearer presentation of the subject we shall here introduce the term "local Reynolds number" as differentiated from the "common Reynolds number." While the latter is based on the kinematic viscosity of the fluid at large, which depends on the prevailing room temperature, the "local Reynolds number" is based on the kinematic viscosity of a certain region in question near the surface of the sphere. Unless specifically stated otherwise, the term "Reynolds number" will be implied to mean the "common Reynolds number."

The results of the drag measurements in the three sets of experiments are given in Tables I to III. The following notation is used in the tables:

A = Cross-sectional area of great circle of sphere,

V = Velocity of undisturbed air,

ρ = Density of air at prevailing room temperature,

R_C = Common Reynolds number,

ν_M = Kinematic viscosity of air at mean layer temperature (arithmetical mean of sphere temperature and room temperature),

ν_C = Kinematic viscosity of air at room temperature,

R_M = Local Reynolds number based on kinematic viscosity at mean layer temperature.

The values of drag coefficient and Reynolds number given in the tables were computed from the measurements of drag, speed, temperature, etc.

The Reynolds number is defined by $R = \frac{LV}{\nu}$, where L is diameter of sphere (5.01″), V has the meaning given above, and ν, the kinematic viscosity of air, is equal to the ratio of the coefficient of viscosity to the mass density.

The drag coefficient is defined by $C_D = \frac{\text{Drag}}{A V^2 \rho/2}$, where the symbols have the meaning given above.

Using the values of drag coefficients and Reynolds numbers given in the tables the curves in Figs. 2 to 8 were plotted.

Figure 2 shows the effect of the wire screen on the flow around the unheated sphere. As may be seen the introduc-

TABLE I.

Drag Measurements without Wire Screen in Tunnel.

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$
Atm. Press.: 760 mm. Hg.; Room Temp.: 31.5° C.; Air Density: .00225 lb./cu. ft.; ν_C: .000173 ft.²/sec.									
28.0	41.07	.260	.133	.515	98.9	31.5	31.5	.000173	98.9
32.0	46.93	.339	.168	.495	113.0				113.0
36.0	52.80	.429	.204	.475	127.2				127.2
40.0	58.67	.530	.240	.455	141.3				141.3
44.0	64.53	.641	.278	.435	155.4				155.4
48.0	70.40	.763	.310	.405	169.6				169.6
52.0	76.28	.896	.337	.375	183.7				183.7
56.0	82.14	1.039	.360	.345	197.8				197.8
60.0	88.00	1.193	.383	.320	212.0	↓	↓	↓	212.0
28.2	41.36	.264	.136	.520	99.6	31.5	31.5	.000173	99.6
32.1	47.08	.341	.169	.495	113.4				113.4
36.0	52.80	.429	.205	.475	127.2				127.2
40.1	58.81	.533	.242	.455	141.6				141.6
42.0	61.60	.585	.259	.440	148.4				148.4
46.0	67.47	.701	.295	.420	162.5				162.5
50.1	73.49	.832	.324	.390	177.0				177.0
54.2	79.50	.973	.349	.360	191.5	↓	↓	↓	191.5
28.1	41.21	.262	.137	.525	99.3	61.0	46.2	.000188	91.4
32.0	46.93	.339	.172	.505	113.0				104.0
40.1	58.81	.533	.250	.470	141.6				130.7
44.2	64.82	.647	.292	.450	156.1				144.0
48.3	70.84	.774	.326	.420	170.6				157.0
52.2	76.57	.903	.353	.390	184.4				170.0
56.0	82.14	1.039	.371	.355	197.8	↓	↓	↓	182.0
Atm. Press.: 760 mm. Hg.; Room Temp.: 26.0° C.; Air Density: .00229 lb./cu. ft.; ν_C: .0001675 ft.²/sec.									
32.0	46.93	.346	.174	.505	116.8	61.0	43.5	.000185	105.7
40.0	58.67	.540	.249	.460	146.0				132.1
44.0	64.53	.653	.291	.445	160.6				145.3
48.0	70.40	.778	.326	.420	175.2				158.5
52.0	76.28	.913	.353	.385	189.8				171.8
56.0	82.14	1.059	.372	.350	204.4	↓	↓	↓	185.0

TABLE I—*Continued.*

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-4}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-4}$
Atm. Press.: 760 mm. Hg.; Room Temp.: 31.5° C.; Air Density: .00225 lb./cu. ft.; ν_C: .000173 ft.²/sec.									
30.0	44.00	.298	.155	.520	106.0	95.0	63.2	.000207	88.5
38.0	55.73	.478	.235	.490	134.2				112.3
42.0	61.60	.585	.278	.475	148.4				124.0
46.1	67.61	.704	.318	.450	162.8				136.0
50.3	73.78	.838	.357	.425	177.7				148.2
54.1	79.36	.970	.386	.400	191.1				159.6
58.1	85.22	1.118	.418	.375	205.3				171.2
Atm. Press.: 760 mm. Hg.; Room Temp.: 31.5° C.; Air Density: .00225 lb./cu. ft.; ν_C: .000173 ft.²/sec.									
34.0	44.87	.383	.190	.495	120.0	43.0	37.2	.000179	116.1
34.0	44.87	.383	.192	.500	120.0	73.0	52.2	.000194	106.0
34.0	44.87	.383	.195	.510	120.0	95.0	63.2	.000207	100.5
38.0	55.73	.478	.227	.475	134.2	47.0	39.2	.000181	127.8
38.0	55.73	.478	.231	.485	134.2	73.0	52.2	.000194	118.7
38.0	55.73	.478	.236	.495	134.2	95.0	63.2	.000207	112.4
44.0	64.53	.641	.284	.445	155.4	49.5	40.5	.000182	148.6
44.0	64.53	.641	.290	.450	155.4	73.0	52.2	.000194	137.5
44.0	64.53	.641	.295	.460	155.4	95.0	63.2	.000207	130.0
50.0	73.34	.828	.330	.400	176.6	47.0	39.2	.000181	167.8
50.0	73.34	.828	.341	.410	176.6	73.0	52.2	.000194	156.1
50.0	73.34	.828	.355	.430	176.6	95.0	63.2	.000207	148.0
54.0	79.21	.966	.353	.365	191.0	45.0	38.2	.000180	182.5
54.0	79.21	.966	.371	.385	191.0	73.0	52.2	.000194	168.8
54.0	79.21	.966	.383	.395	191.0	95.0	63.2	.000207	159.5

tion of the screen into the tunnel causes the drag curve to be shifted toward smaller Reynolds numbers.

Figures 3 to 5 show the characteristic curves for the three different conditions of the approaching air stream. In each figure curves are given for the unheated sphere, as well as for the sphere at a few higher temperatures. The temperature of the sphere is indicated along each curve. The Reynolds numbers in these figures are common Reynolds numbers, based on the kinematic viscosity at the prevailing room tem-

perature, irrespective of the sphere temperature. The room temperatures are given in each figure.

Figures 6 to 8 are plotted from values computed from the

TABLE II.

Drag Measurements with Wire Screen 20" in Front of Sphere.

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.5° C.; Air Density: .00230 lb./cu. ft.; ν_C: .000167 ft.²/sec.									
24.0	40.19	.254	.112	.440	100.3	25.5	25.5	.000167	100.3
28.0	46.93	.347	.141	.405	117.1				117.1
32.0	53.83	.456	.168	.370	134.3				134.3
36.0	60.43	.575	.190	.331	150.8				150.8
38.0	63.80	.641	.202	.315	159.2				159.2
40.0	67.47	.716	.214	.300	168.3				168.3
42.0	70.40	.780	.228	.290	175.6				175.6
44.0	74.51	.874	.247	.285	185.9				185.9
22.0	36.67	.212	.098	.460	91.5	25.5	25.5	.000167	91.5
24.0	40.19	.254	.112	.440	100.3				100.3
26.0	43.41	.296	.127	.430	108.3				108.3
28.0	46.93	.347	.142	.410	117.1				117.1
30.0	50.60	.403	.155	.385	126.2				126.2
32.0	53.83	.456	.169	.370	134.3				134.3
34.0	57.20	.515	.180	.350	142.7				142.7
40.0	67.47	.716	.212	.295	168.3				168.3
42.0	70.40	.780	.226	.290	175.6				175.6
22.0	36.67	.212	.099	.470	91.5	61.0	43.2	.000185	82.6
24.0	40.19	.254	.116	.455	100.3				90.5
26.0	43.41	.296	.130	.440	108.3				97.8
28.0	46.93	.347	.147	.425	117.1				105.7
30.0	50.60	.403	.161	.400	126.2				114.0
34.0	57.20	.515	.187	.365	142.7				128.8
36.0	60.43	.575	.197	.340	150.8				136.1
38.0	63.80	.641	.206	.320	159.2				143.7
40.0	67.47	.716	.218	.305	168.3				151.9
42.0	70.40	.780	.231	.295	175.6				158.5
22.0	36.67	.212	.101	.475	91.5	84.0	54.8	.000197	77.6
24.0	40.19	.254	.119	.465	100.3				85.0
28.0	46.93	.347	.150	.430	117.1				99.3
30.0	50.60	.403	.165	.410	126.2				107.0
32.0	53.83	.456	.182	.400	134.3				113.9
34.0	57.20	.515	.194	.375	142.7				121.0
36.0	60.43	.575	.205	.355	150.8				127.8
38.0	63.80	.641	.217	.340	159.2				134.9
40.0	67.47	.716	.229	.320	168.3				142.7
42.0	70.40	.780	.241	.310	175.6				148.9

TABLE II—*Continued.*

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.5° C.; Air Density: .00230 lb./cu. ft.; ν_C: .000167 ft.²/sec.									
22.0	36.67	.212	.102	.480	91.5	106.0	65.8	.000209	73.1
24.0	40.19	.254	.120	.470	100.3				80.1
26.0	43.41	.296	.135	.455	108.3				86.6
28.0	46.93	.347	.152	.440	117.1				93.6
32.0	53.83	.456	.186	.410	134.3				107.3
34.0	57.20	.515	.200	.390	142.7				114.1
36.0	60.43	.575	.216	.375	150.8				120.5
38.0	63.80	.641	.226	.355	159.2				127.2
40.0	67.47	.716	.237	.330	168.3				134.5
42.0	70.40	.780	.248	.320	175.6	↓	↓	↓	140.4
24.0	40.19	.254	.115	.455	100.3	61.0	43.2	.000185	90.5
24.0	40.19	.254	.118	.465	100.3	84.0	54.8	.000197	85.0
32.0	53.83	.456	.174	.380	134.3	61.0	43.2	.000185	121.2
32.0	53.83	.456	.180	.395	134.3	84.0	54.8	.000197	113.9
32.0	53.83	.456	.186	.405	134.3	106.0	65.8	.000209	107.3
40.0	67.47	.716	.220	.305	168.3	61.0	43.2	.000185	151.9
40.0	67.47	.716	.229	.320	168.3	84.0	54.8	.000197	142.7
40.0	67.47	.716	.238	.332	168.3	106.0	65.8	.000209	134.5

same measurements as used in Figs. 3 to 5, but the Reynolds numbers here given are local Reynolds numbers based on the kinematic viscosity corresponding to the arithmetical mean of room and sphere temperature. This might also be called "mean layer temperature," assuming linear temperature variation in the boundary layer.

The results of the heat loss measurements at various air speeds and different screen positions, for a number of temperature differences between sphere and room, are plotted in Fig. 9. Although these measurements are not very exhaustive the curves seem to indicate that the more turbulent the flow reaching the sphere, the greater the quantity of heat which it carries away. This is really to be expected, since an eddying air stream means greater mixing of parts and reduces the thickness of the conducting film near the surface, thereby increasing the temperature gradient. Figure 9 is here inserted because the knowledge of the heat losses may prove important

toward the solution of the final problem. They may also be used for the determination of the thickness of the boundary layer. The values of heat losses given in this figure include

TABLE III.

Drag Measurements with Screen 15" in Front of Sphere.

Manometer Reading.	Air Speed F.P.S.	$AV^2\rho/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 26.0° C.; Air Density: .00229 lb./cu. ft.; ν_C: .0001675 ft.²/sec.									
26.0	39.60	.246	.104	.420	98.5	26.0	26.0	.0001675	98.5
28.0	42.53	.284	.116	.410	105.8				105.8
30.0	45.47	.324	.128	.395	113.1				113.1
32.0	48.40	.368	.140	.380	120.4				120.4
34.0	51.33	.413	.150	.365	127.7				127.7
36.0	54.56	.467	.161	.345	135.7				135.7
38.0	57.79	.524	.171	.325	143.8				143.8
40.0	61.31	.590	.182	.310	152.5				152.5
42.0	64.24	.648	.194	.300	159.8				159.8
44.0	67.17	.708	.204	.290	164.6				164.6
48.0	73.48	.847	.239	.280	182.8				182.8
50.0	76.71	.923	.258	.280	190.9	↓	↓	↓	190.9
26.0	39.60	.246	.104	.420	98.5	26.0	26.0	.0001675	98.5
30.0	45.47	.324	.127	.390	113.1				113.1
34.0	51.33	.413	.150	.360	127.7				127.7
38.0	57.79	.524	.170	.325	143.8				143.8
42.0	64.24	.648	.192	.295	159.8				159.8
46.0	70.25	.774	.221	.285	174.8				174.8
50.0	76.71	.923	.256	.280	190.9	↓	↓	↓	190.9
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.0° C.; Air Density: .00230 lb./cu. ft.; ν_C: .0001665 ft.²/sec.									
26.0	39.60	.247	.107	.430	99.1	61.0	43.0	.000185	89.2
30.0	45.47	.326	.132	.405	113.8				102.4
34.0	51.33	.415	.155	.375	128.5				115.6
38.0	57.79	.526	.175	.335	144.6				130.1
42.0	64.24	.650	.197	.305	160.8				144.7
46.0	70.25	.777	.224	.290	175.8				158.2
50.0	76.71	.926	.260	.280	192.0	↓	↓	↓	172.8
26.0	39.60	.247	.110	.445	99.1	84.0	54.5	.000197	83.8
30.0	45.47	.326	.137	.420	113.8				96.2
34.0	51.33	.415	.160	.385	128.5				108.6
38.0	57.79	.526	.181	.345	144.6				122.2
42.0	64.24	.650	.202	.310	160.8				135.9
46.0	70.25	.777	.226	.290	175.8				148.6
50.0	76.71	.926	.262	.285	192.0	↓	↓	↓	162.2

TABLE III—*Continued.*

Manometer Reading.	Air Speed F.P.S.	$A V^2_p/2$ Lb.	Drag Lb.	Drag Coeffic.	$R_C \times 10^{-3}$.	Sphere Temp. ° C.	Mean Layer Temp. ° C.	ν_M ft.²/sec.	$R_M \times 10^{-3}$.
Atm. Press.: 758 mm. Hg.; Room Temp.: 25.5° C.; Air Density: .00230 lb./cu. ft.; ν_C: .000167 ft.²/sec.									
26.0	39.60	.247	.113	.455	98.8	106.0	65.8	.000209	79.0
30.0	45.47	.326	.142	.435	113.4	↓	↓	↓	90.7
34.0	51.33	.415	.166	.400	128.1				102.4
38.0	57.79	.526	.190	.362	144.2				115.2
42.0	64.24	.650	.210	.325	160.3				128.1
46.0	70.25	.777	.232	.300	175.3				140.1
50.0	76.71	.926	.268	.290	191.4				153.0
26.0	39.60	.247	.107	.435	98.8	61.0	43.2	.000185	89.2
26.0	39.60	.247	.110	.445	98.8	84.0	54.8	.000197	83.8
26.0	39.60	.247	.113	.460	98.8	106.0	65.8	.000209	79.0
34.0	51.33	.415	.156	.375	128.1	61.0	43.2	.000185	115.6
34.0	51.33	.415	.160	.385	128.1	84.0	54.8	.000197	108.6
34.0	51.33	.415	.166	.400	128.1	106.0	65.8	.000209	102.4
42.0	64.24	.650	.197	.305	160.3	61.0	43.2	.000185	144.7
42.0	64.24	.650	.203	.315	160.3	84.0	54.8	.000197	135.9
42.0	64.24	.650	.211	.325	160.3	106.0	65.8	.000209	128.1
50.0	76.71	.926	.261	.280	191.4	61.0	43.2	.000185	172.8
50.0	76.71	.926	.264	.285	191.4	84.0	54.8	.000197	162.2
50.0	76.71	.926	.269	.290	191.4	106.0	65.8	.000209	153.0

radiation as well as convection losses, but the former constitute only a small percentage of the total heat flow, amounting to less than 15 per cent even at the maximum temperature difference investigated.

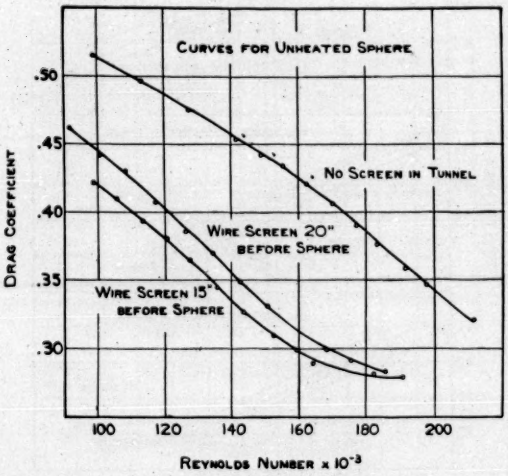
6. DISCUSSION OF RESULTS.

As may be seen from Fig. 2 the introduction of the wire screen in front of the sphere served its intended purpose; namely, it caused a shift of the drag curve toward smaller Reynolds numbers. This is in accordance with the findings of other investigators (4). Moving the screen from 20 inches to a position 15 inches in front of the sphere did not change the flow greatly.

In Figs. 3 to 5 the effect of heating of the sphere is clearly seen. The curves in Fig. 5 might be interpreted as represent-

ing a region to the right of that covered by Fig. 3, while the curves in Fig. 4 fit in somewhere between these two sets. The curves show that raising the temperature of the sphere above that of the approaching air definitely shifts the drag curve. This change must be attributed to the changed properties of the boundary layer, since as we have seen, the heat flowing from the sphere affects only this thin layer, disregarding the relatively unimportant effect of the radiant heat. Our curves show that when the sphere is heated the drag coef-

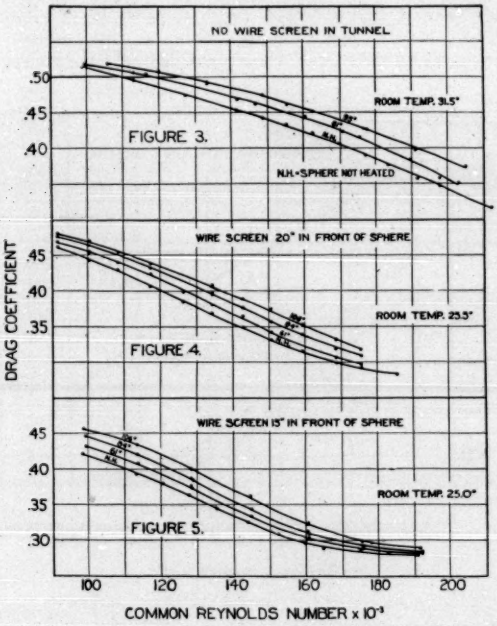
FIG. 2.



ficient at a given common Reynolds number is increased. But this is just the opposite of what happens when the air stream around the sphere is made more turbulent, as may be seen from Fig. 2. So we may say that as a result of the heat passing from the sphere into the fluid the flow becomes less turbulent, thus causing the point of separation of the boundary layer from the surface to move forward and the drag to increase. (See reference (5) for a discussion of the point of separation.)

In order to understand why increased temperature of the boundary layer should cause the flow to become smoother, we need only remember that increased temperature means increased air viscosity and decreased air density, and both

FIGS. 3, 4 AND 5.



these changes contribute toward increasing the kinematic viscosity and hence are conducive to smoother flow.

We can explain the shifting of the drag curves caused by the heating of the sphere in a more general way. On the basis of the principle of dynamic similarity, all points in the drag coefficient—Reynolds number diagram should lie on a single curve, provided that the Reynolds number values

FIGS. 6, 7 AND 8.

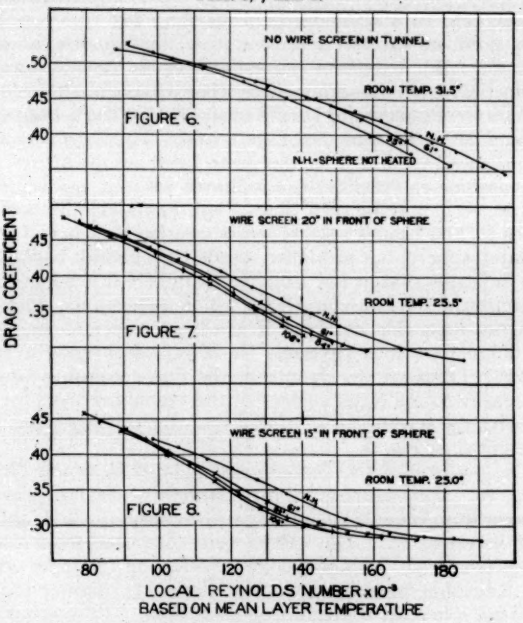
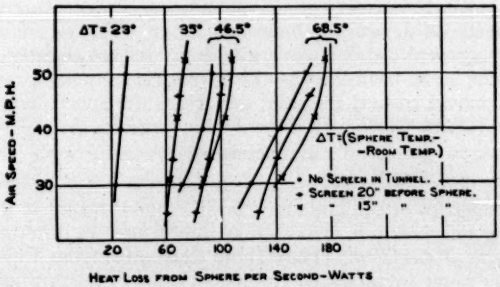


FIG. 9.



used are representative of the flow. Obviously then for the present case of a non-homogeneous fluid the common Reynolds numbers are not representative, but there exist some local Reynolds numbers corresponding to certain laminae within the heated boundary layer that are representative.

As a first step toward the determination of the flow-governing laminae we have plotted Figs. 6 to 8. The Reynolds numbers in these are based on the mean layer temperatures. In Fig. 6, which represents the conditions without any screen in the tunnel, it may be seen that at low Reynolds numbers the curves for the heated sphere nearly overlap the curve for the unheated sphere, but at higher speeds they branch to the left. This indicates that at low Reynolds numbers it is a layer near the center of the boundary layer which governs the flow, but at higher Reynolds numbers it is a layer closer to the outer film boundary which governs. In Figs. 7 and 8 the trend is similar. These curves show that the flow-governing layer is even closer to the outer surface of the boundary layer for the flow with the screen in the tunnel than for the smoother flow represented in Fig. 6.

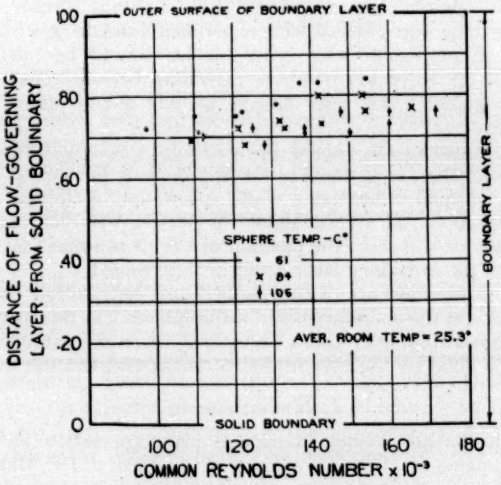
The position of the flow-governing layer is clearly shown in Fig. 10. This figure gives the distances of the flow-governing layer from the solid boundary within the Reynolds number range investigated. The values were computed from curves in Figs. 4 and 5. This was done by selecting the layer whose local Reynolds number equals the Reynolds number for the unheated sphere at a given drag coefficient. In making this computation use is made of the relation between temperature and kinematic viscosity of air, and a linear temperature variation assumed across the boundary layer. The figure shows that in general the flow-governing layer moves slightly away from the solid boundary as the Reynolds number increases, and it moves toward the solid surface as the sphere temperature is raised.

7. CONCLUSIONS.

The foregoing experiments confirm the proposition that the properties of the boundary layer around a solid in a fluid moving relative to it are of great importance in determining the type of the flow. They show that within the Reynolds number range investigated (100,000–170,000), the flow around

the sphere is governed by the properties of a lamina within the boundary layer, approximately seventy-five-hundredths of the boundary layer thickness away from the solid surface. The distance of this lamina varies slightly with the Reynolds number. It is closer to the surface for small Reynolds numbers, and moves farther out as the Reynolds number increases.

FIG. 10.



The position of the flow-governing layer seems to depend to a slight degree on the temperature of the sphere, and also on the relative turbulence of the approaching flow.

8. CONCLUDING REMARKS. SUGGESTIONS FOR FURTHER EXPERIMENTS.

The experiments described in the foregoing pages were carried out over a limited range of Reynolds numbers. They were also confined to a sphere at temperatures higher than the air stream. It is difficult to say whether the effects of the boundary layer, at Reynolds number beyond the range investigated, are of the same order as here found; whether in

fact they are of any importance at all once the flow is quite turbulent. It was pointed out at the beginning of this paper that a knowledge of the causes for the instability of the laminar flow in the boundary layer is essential for the solution of the problem. Our experiments show that an outward diffusion of heat energy serves to stabilize the existing flow. Possibly by cooling of the solid greater instability will be brought about and turbulence promoted.

To answer these questions and to obtain further data along these lines the following experiments should be suitable:

1. Experiments with heated solids should be made in regions of Reynolds numbers extending beyond those here investigated. The tests should include solids other than spheres.

2. Experiments should be made on a sphere cooled to temperatures below those of the approaching air.

3. Instead of heating a whole sphere only a circular zone normal to the air stream should be heated. By varying the location of this strip the position of the most important section of the boundary layer could be determined.

4. Measurements of the temperature distribution should be made in the boundary layer of the sphere. In this way the flow-governing layer could be located more accurately than can be done on the assumption of linear temperature variation.

9. ACKNOWLEDGMENT.

The author wishes to express his appreciation for the encouraging and helpful guidance in this work to Dr. Max M. Munk, sponsor of this dissertation, and for valuable practical suggestions rendered by Professor L. H. Crook.

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NEW METHODS OF PHYSICAL INVESTIGATION OF TONE QUALITY.

BY

HARL MACDONALD and O. HUGO SCHUCK.

INTRODUCTION.

In considering any musical tone the musician is, of course, interested in its pitch, but more particularly in its quality. The quality of a musical tone is fundamentally a subjective consideration. It is the mental and emotional response of the listener to the tone and, as such, is entirely personal. However, most people of a particular race agree on the psychological effects of certain tones. They agree, for instance, on the differences between the effects of a violin tone and a flute tone, so that there is now, and has been for centuries, general agreement as to the qualities of the tones of musical instruments.

Recently scientific investigators have interested themselves in the subject with the result that they have determined the characteristics of sound waves, and have tried to find the correlation between the physical, objective properties of sound, and the psychological, subjective impression of quality. The various experiments begun in the last century by Helmholtz and Rayleigh confirmed the opinions held by musicians concerning the relations between fundamental and overtones, and further investigation has proved conclusively that every musical tone consists of a fundamental and a number of overtones whose frequencies of vibration are higher than that of the fundamental frequency. It has been shown that the quality of a tone depends on the relative strengths of the particular overtones, and that the ear recognizes a particular tone quality by analyzing the tone into its fundamental and overtone frequencies and responding to each separately. This is usually done automatically and unconsciously, though a very few people can do it consciously.

We see then that the first step in the objective study of quality is to determine the relationship between the physical

constitution of the sound wave and the listener's perception of its quality. We must take, for example, a flute tone, which we recognize by ear as a flute tone, and measure the loudness of its fundamental and of each of its overtones which lie within the hearing range of the ear. By using hollow resonators tuned to the various frequencies, Helmholtz and Rayleigh obtained qualitative results. More recently Miller obtained quantitative results by taking oscillograms of the sounds, similar to much enlarged phonograph-record tracks, and determining the strengths of the various components through harmonic analysis of the oscillograms. It will be recognized that this method, while accurate, is tedious and time consuming, since after an oscillogram is made, it must be developed, enlarged and analyzed before the composition of the sound is known. Unfortunately, it has been the only method available for analyzing sounds of short duration such as musical tones produced by human players. When the tones may be sustained and held constant for some length of time, as those from organ pipes, various electrical methods of analysis have been available, but they are not of much value in analyzing tones of changing quality and short duration such as are produced by human means, either instrumental or vocal.

THE SOUND PRISM.

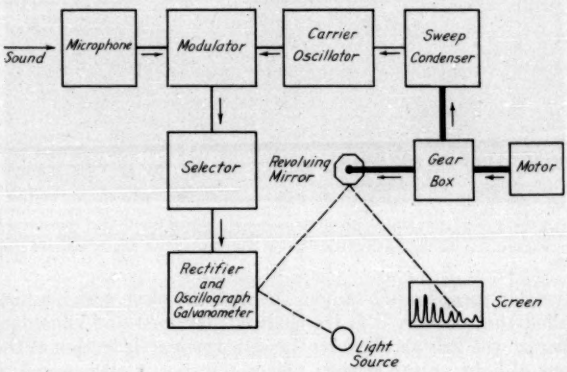
There has recently been developed by one of the authors at the Moore School of Electrical Engineering of the University of Pennsylvania an electrical device for making a practically instantaneous analysis of a sound and continuously showing its frequency spectrum on a screen. With this device, which has been named the "Sound Prism," the observer (or the player himself) can hear the tone and can observe its spectrum at the same time. Tones of short duration may thus be analyzed, and during the production of the tone changes in the spectrum can be followed by eye as the ear hears them as changes in quality. Photographic records may also be made at any desired instant during the duration of the tone.

The Sound Prism is essentially an automatically operated heterodyne wave analyzer such as is now well known, but it is unique in incorporating two new features. First, instead of the carrier frequency being slowly changed, it is rapidly swept

through its range a number of times a second. This has necessitated the development of a special form of filter to enable operation at such a high rate of frequency change. Second, the indications of the measuring device are arranged to produce vertical motion of a spot of light on a screen while the spot is moving horizontally in proportion to the carrier frequency, so that the spot traces the frequency spectrum on the screen. The tracing of the spectrum is continuously repeated at a rate rapid enough to enable the eye, through its property of persistence of vision, to see the path of the spot as a steady line of light on the screen. This method of obtaining a visual picture of the relationship between two quantities has been described elsewhere.[1]

The simplified functional diagram of Fig. 1 will serve to illustrate the operation of the Sound Prism. Sound vibra-

FIG. 1.

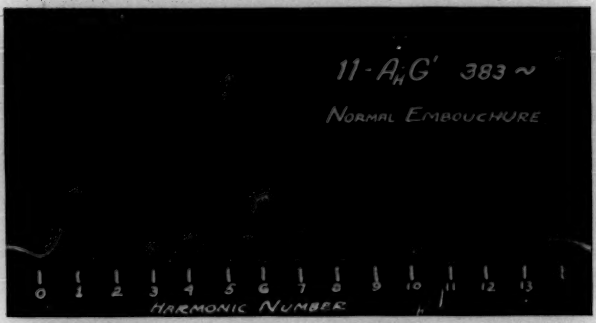


tions are picked up by the microphone; the resulting electrical currents are amplified and led to the balanced modulator where they are combined with the constant amplitude carrier current generated by the carrier oscillator to give as products of modulation the upper and lower first order side-bands.

[1] O. H. Schuck, *Proc. I. R. E.*, 20, 1580, 1932.

Since the frequency generated by the carrier oscillator is controlled by the position of the motor-driven revolving sweep condenser, the carrier frequency is continually changing, decreasing from an upper limit to a lower limit during a half revolution of the condenser shaft. All of the side-band components experience a corresponding continual change in frequency, though they maintain constant their amplitudes and their frequency differences from the carrier as determined by the amplitudes and frequencies of the respective components of the sound. As the frequency of each side-band component

FIG. 2.



A sample sound prism analysis record: the analysis of a clarinet tone G^1 (383) normal embouchure. The harmonics are numbered, 1 being the fundamental, 2 the first partial, and so on. It will be noticed that the third harmonic is absent, and that the second and fifth are especially prominent.

passes in turn through the pass band of the filter, which is here called the selector, it is transmitted, rectified and caused to operate the galvanometer to cause a vertical deflection of the spot of light on the screen. Since the spot is also caused to move horizontally across the screen by the revolving mirror geared to the condenser shaft, the transient response curve of the selector is drawn for each side-band component, thus tracing on the screen a line having a number of peaks, one for each component of the sound. The height of each peak is proportional to the amplitude of the respective side-band component and thus proportional to the amplitude of the cor-

responding component of the sound; its position is dependent upon the frequency of the carrier at the time the respective side-band passes through the pass band of the selector. This occurs when the difference between the carrier frequency and the pass-band frequency is equal to the frequency of the corresponding component of the sound. Instead of the horizontal scale being graduated in terms of carrier frequency it is more conveniently graduated in terms of the difference, so that the horizontal position of the peak may be referred to the frequency of the particular component of the sound which it represents. The vertical and horizontal positions of the peaks of the curve traced on the screen therefore give the amplitudes and frequencies of the components of the sound, that is, they make up the frequency spectrum of the sound reaching the microphone.

The great advantage of the use of the Sound Prism for musical tone analysis is its rapidity of action, enabling more lengthy and exhaustive studies to be undertaken than have heretofore been feasible. Its ability to indicate any frequency makes it possible to measure inharmonic as well as harmonic overtones, thus enabling analysis of such tones as bell tones and of the playing noise which is inseparable from the tones of certain instruments. Since the operation of the Sound Prism is instantaneous so far as the eye can see, it is possible to observe and study the changes in quality which occur while the tone is being played. A direct correlation between the objective analysis and the subjective response is thus made possible for the first time.

Some brief investigations have been made with the original laboratory model and some interesting results have been obtained.

The work already under way has been in the field of the stringed instruments with regard to variations in the relative value of harmonics with changing direction of the bow; analysis of the vibrato; the so-called "wolf"; and the breaks in the scale produced by the four strings.

In another field, that of the human voice, lie many interesting possibilities. Preliminary work would indicate that the greatest results will be obtained through an analysis of the skull-resonances of individual voices and a study of corrective

technique where desirable resonances are missing or in the case of those too pronounced in certain ranges.

The University of Pennsylvania has authorized the building of a new and improved model of the Sound Prism under a Rockefeller Foundation grant to Mr. Knox McIlwain, of the Moore School of Electrical Engineering. The new instrument will be more compact, rugged and portable than the original laboratory model, its range and accuracy will be greater and its operation will be simplified so that any musician can operate it himself. This instrument is now nearing completion and will be ready for use in the fall of the year. Comprehensive plans have been laid for further investigations on musical tones, and the results will be presented in detail in subsequent publications.

THIS SUBJECT OF ERROR.

BY

L. E. DODD, Ph.D.,

University of California at Los Angeles.

In any series of measurements, however carefully conducted, the resulting "most probable value" of the desired quantity is not very likely to be the true value. The chances are decidedly against it. Probably not nearly 100 per cent. of the many who work in science in one way or another—teacher, student, laboratory researcher, engineer—have become sufficiently appreciative of the fact that the most probable value, even though it may be more probable than any other value, is itself not very probable. In an article [1] published last year, Eddington emphasizes this, incidental to other points, as to least squares, that invite detailed study by scientific workers generally.

When we look up in our more recent tables of physical constants, the value of say the ice point, and read 273.18 $\pm$ 0.03° K., just what does this mean to us? The printed value 273.18 we would naturally regard as correct, and we would use it for substitution in formulas. Just what does the 273.18 mean? The "most probable value," of course. This term, however, charms too easily, and we are inclined to commit ourselves to its keeping with a faith that in quality comes much too near that placed by people of former days in cheap thermometers. When pinned down to it we will admit that the 273.18 is not necessarily absolutely correct, but being the "most probable value" it is, we feel, the particular value safest to depend upon. But after all, *how* safe is it?

Eddington insists that the purpose of the theory of combination of observations is not to find the most probable value. The object of the theory is "to assist scientific investigation and not to answer mathematical conundrums."

[1] "Notes on the Method of Least Squares," *Proceedings of the Physical Society*, March 1, 1933, pp. 271–287.

Also, is it not too common practice to regard 273.18 as *correct within the limits* $(273.18 + 0.03) = 273.21$ and $(273.18 - 0.03) = 273.15$? This wrong point of view however means that the person holding it, as Eddington says, thereby tacitly admits "his habit is to tell the truth only on alternate occasions." The exact meaning of the 273.18 ± 0.03 is always to be kept in mind. It is a 50–50 risk to suppose the true value to be within the indicated limits, 273.21 and 273.15. But arbitrarily and to a considerable degree unconsciously, do we not often tend to weight the 50 that stands for the *within*, at least 50 per cent. more than the 50 that stands for the without!

What we determine and can place practical dependence upon in any carefully repeated measurement where an average value is obtained, is a *value-range* having specified limits, together with the amount of *risk* we run in regarding the true value as being located inside this range. More specifically, what is of practical importance to us is, ideally, the "range per risk." The most reliable determination then would be that with the lowest range per risk. The ideal criterion of "range per risk" is subject to some ambiguity. As the most useful general criterion of accuracy (that is, reliability) therefore Eddington adopts the mean square error, believing it to be a criterion almost as good as the ideal.

The probable error gives us the 50–50, or 1 in 2, risk. Doubling the probable error gives a 1 in 6 risk, but "a navigator determining his position at sea would not be satisfied to run his ship on the rocks once in six times!" If we take $2\frac{1}{2}$ times the probable error, our risk is 1 in 10, "probably not greater than the risk I habitually take in drawing inferences from observational evidence."

Briefly, the practical result of any measurement on a physical quantity is a range of values rather than a particular determined value upon which great reliance can be placed. This range of values cannot be regarded with certainty as containing the true value, for after all our efforts the mustang may still not be in the corral. Where the risk is not too large, the animal certainly *ought* to be, if not inside, at any rate in the more immediate vicinity—but we are compelled by the truth to say *ought*, and not *must*.

Three additional quotations from the paper can only emphasize its interest and cogency: "It is not a crime to fail to reach the highest possible accuracy; it is a crime to fail to reach the accuracy which you claim." "The admissible risk varies enormously with the instrument. In testing clinical thermometers you must consider what proportion of users you are prepared to kill by misleading the doctor. In testing taximeters you may justifiably take a somewhat higher risk of permitting overcharge." "Effectively then we have replaced a mass of complicated observational material by the *best possible equivalent single direct observation* of the quantity x that we are seeking. The task of the theory of the combination of observations ends with this reduction."

Finally, the present writer would caution against a possible misunderstanding. Calling attention to the general need for a more careful attitude toward accepted values of physical constants as they appear in print at the present time, by no means disparages a published list such as that of Professor Birge,[2] where the range in each case is given, the risk being 50 per cent., as far as data from different sources were deemed sufficient to warrant these estimates. To the writer, ever since the publication of this comprehensive list, it has always seemed—and the feeling continues—that too much credit can hardly be given its author for the extensive, careful and difficult work accomplished in its preparation. It will doubtless need revising as time brings new and more satisfactory evidence. As Birge says, "The most probable value of today is not that of tomorrow, because of the never-ending progress of scientific research." The very existence, however, of such a coherent list cannot but prove an additional stimulus to research workers, not only to improve technique, but what is equally important, to specify in their publications the precise and complete details bearing on error calculations and estimates.

[2] "Probable Values of the General Physical Constants," *Physical Review Supplement*, July, 1929, pp. 1–73.

Goldenrod Rubber.—Experiments by the U. S. Department of Agriculture show that goldenrod is likely to become the most satisfactory source of domestic rubber. Rubber of good quality has been made from this plant but the details of extraction and manufacture have not been developed sufficiently to produce the material on a commercial scale. With present knowledge it would not be possible to make rubber from goldenrod at the prevailing prices of about 15 cents a pound.

L. G. POLHAMUS, of the Department, has been engaged in a study of rubber plants for several years. So far, *Solidago leavenworthii* has the highest rubber content of any of the goldenrods analyzed. Specimens have produced more than 12 per cent. rubber. Department officials hasten to emphasize that experiments have not progressed far enough to warrant anyone going into the business of growing or collecting goldenrod to sell for making rubber. Also, the Department has no goldenrod plants for distribution.

Apparently it will not be easy to domesticate the plant. Once it is cultivated, its vulnerability toward insect and fungous enemies increases considerably. Plants that grew upright in the wild state, show a tendency to fall over under the influence of more favorable growing conditions. What is more, one group of plants has exhibited practically no response to fertilizers. Breeding experiments have advanced to the point where the outward characteristics of the plant may be controlled, but how to get a maximum yield of rubber is still one of Nature's secrets. Genetically identical plants gave a rubber content ranging from 1.07 to 3.55 per cent.

Other promising sources of rubber are the guayule, a shrub which grows well in the southwest, and Hevea, the rubber tree of the tropics. Several thousand acres have been planted to guayule in California by a company interested in its development. For maximum yields, the plant is harvested only every fourth year. Because of this, a large area and a succession of plantings to insure a continuous harvest is necessary. At the present, the Department has 30,000 Hevea trees growing in Florida. It is believed they will yield as well as those in the East Indies.

There are two general processes for extracting rubber from plants. With the mechanical method, used on guayule, the plants are ground up and macerated in a pebble mill. The rubber gathers into small lumps which can be separated from the powdered woody material by flotation in water. The other method is to dissolve the rubber out of the plants by chemical solvents. This is the method best suited to goldenrod.

C.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

GRADUATION OF PRECISION CIRCLES.

The Bureau's circular dividing engine and the type of graduating work done on it have been briefly described in Technical News Bulletin nos. 128 and 149 (December, 1927 and September, 1929). This instrument has been used during the past nine months in producing graduated circles of a high order of accuracy, for the U. S. Coast and Geodetic Survey.

The triangulation program of the Coast and Geodetic Survey has necessitated the acquiring of a considerable number of graduated circles for new theodolites. Two types of Parkhurst theodolites, for which circles have been supplied, are employed in this program, the first order theodolite using a 9-inch circle, and the second order theodolite using a 6½-inch circle. Both sizes of circles have been made and graduated at the National Bureau of Standards. In addition to the new circles, a number of old ones have been regraduated.

Nearly all of the new circles are of cast sterling silver. A few circles are of duralumin and a few, for regraduation, are of bronze with an inserted sterling silver ring. These special circles require no particular discussion at this time.

The circles go through the following processes: (1) Casting of the sterling-silver disk; (2) heat-treatment to relieve the casting strains; (3) machining to approximate size; (4) heat-treatment to relieve machining strains and to further stabilize the material; (5) machining to a size suitable for plating; (6) electroplating with silver, using cyanide bath; (7) machining to finished size; (8) inscribing the coarse series of numbers and graduations; (9) polishing to a plane on pitch-lap with chromium oxide; (10) inscribing the fine numbers; (11) polishing the burrs from the fine numbers; (12) graduating the circle; (13) polishing the burrs from the graduation-lines; and (14) filling the lines with special black ink.

With the exception of a few of the first circles, the entire

* Communicated by the Director.

task from the casting of the silver to the completion of the graduated circle was carried out at this Bureau.

In the heat treatment to relieve casting strains, the disks are placed in an electric muffle furnace, and the temperature slowly raised to between 550 and 600° C. with the disks in a reducing atmosphere of illuminating gas. This temperature is held for 2 hours and the disks are then allowed to cool in the furnace. The second heat treatment, given after the disks are machined nearly to size, is similar to the first one, except that the temperature is only 400° C. and the time 1½ hours. In this second treatment each disk is mounted on a special mandrel so as to be supported as uniformly as possible. The distortion of circles by internal strains is believed to be one cause in the past for what have been regarded as incorrectly-graduated circles. These two heat treatments are for the purpose of obtaining as great dimensional stability as possible.

The silver-plating was found necessary because it was not possible to graduate the sterling-silver disks and obtain the kind of graduation-line required, for example, on rolled silver. The crystals of the sterling silver are large, and differ in physical characteristics even in a single circle so that the graduating tool will not cut uniform lines all around the circle. Many of the lines made on the sterling-silver crystals seem to be torn rather than cut.

The polishing of the circle is an essential part of the present procedure. This is not done to produce a reflecting surface but for the purpose of getting a true plane for two reasons, first because the accuracy required demands it and secondly because lines with smooth, sharp edges can be made only on such a surface.

In the actual graduation of the circles several changes and improvements have been made, perhaps the most important of which is that of using a specially constructed diamond-point, adjusted so that a very fine shaving is cut out of the material, leaving sharp edges to the line and a very small amount of burr which is quickly removed with a pitchlap and chromium oxide. No charcoal is used in the process at any time.

Although several changes in the ruling mechanism of the dividing-engine were found necessary, the principal elements

of the machine, the main central bearing, the worm, the gear-teeth around the periphery of the central table, and the contour of the correcting-device, were found to be satisfactory.

After the circles have been completed they are checked on the Bureau's circle-testing machine. Because of the limited time a complete test of the circle cannot be made, but with the microscopes set 90° apart it is possible to determine with fair accuracy whether or not the graduations are uniformly spaced.

The Coast and Geodetic Survey then makes a more extended test of the circles when actually mounted in a theodolite. A series of collimators set up around a room with the theodolite in the center enables measurements to be made just as they would be in the field, sighting many miles away. The results obtained in these tests agree with those made by this Bureau within the range of experimental error.

There is considerable difficulty in stating precisely how accurate these circles are. When it is considered that there are 4320 lines on each of these circles and that on these 9-inch circles an angle of one second is about 0.00002 inch, one must have considerable numerical data to support any statement made. These circles certainly are accurate within the tolerance of two seconds of arc given in the specifications. It is believed that they may be within approximately ± 1 second.

The successful carrying out of this program of preparation, graduation, and calibration of precision-circles at the National Bureau of Standards has involved the fullest coöperation of several technical divisions of the Bureau, particularly the divisions of metallurgy, chemistry, optics, and weights and measures. L. V. Judson, chief of the length section, has had the immediate supervision of this program and B. L. Page has carried out the actual graduation of these circles.

A paper giving more detailed technical description of this work is in preparation.

SILVER IN CHEMICAL EQUIPMENT.

American silver producers recently established a research associateship at the Bureau to make a survey of the existing and potential uses of silver as an engineering material. The chemical industries have for some time employed silver and

silver-lined equipment for certain processes. A brief review of such uses of silver is given in the present paper and mention is made of a few suggested potential uses in similar fields.

Strong alkalies which have a vigorous action upon base metals and refractories can be handled readily in pure silver. Sodium and potassium hydroxides can be melted and cast in silver vessels without appreciable contamination of the product or attack upon the containers and molds.

Silver equipment has been used where dilute hydrochloric acid was one of the products but cannot be considered as suitable for general use in the manufacture of this material. On the other hand, chlorine, which also attacks silver, can be handled in silver apparatus in both the wet and dry conditions because of the adherent coating of silver chloride which is not soluble under these conditions.

Organic acids and compounds can for the most part be handled successfully in silver equipment. Acetic acid, for example, does not attack silver even at the point where the hot vapors condense during distillation. Glycerine which is intended for pharmaceutical use has sometimes been shipped in steel barrels lined with silver.

The use of silver in the manufacture of gelatin and food products such as jams and fruit juices is not unusual abroad. White vinegar intended for use in pickling can be made without danger of discoloration in silver equipment.

FINENESS OF MOLDING SAND.

A knowledge of the fineness of the size distribution or the constituent particles of a molding sand is decidedly important. No difficulty is encountered in sizing relatively large particles with sieves; however, for particles of smaller diameter than 50 microns, other methods must be employed. Recent work in the experimental foundry of the Bureau has shown that a modification of the sedimentation method can be readily applied in such cases. By means of a transfer pipette a sample is taken of a water suspension of the molding sand which has been allowed to settle for a definite period of time which depends upon the size separation desired. The weight of this sample, after being evaporated to dryness, affords a basis for

calculating, on a percentage basis, the size distribution of that portion of the original material which is too fine to be classified by sieve analysis. The method can be simplified somewhat for use as a rapid method suitable for clay substance determinations in foundry control work and routine testing. For the determination of the fineness of molding sands the pipette method surpasses other available methods in ease of manipulation and scope of information obtained.

VACUUM FUSION METHOD FOR DETERMINATION OF GASES IN METALS.

The applicability of the vacuum fusion method for the determination of oxygen, nitrogen, and hydrogen in metals, has been established for plain carbon steels but not, as yet, for alloy steels which are so important in modern metallurgy. The difficulty in the field of alloy steels has resulted from the lack of reliable auxiliary methods of analysis to confirm the results obtained by the vacuum fusion method.

It is believed, however, that two methods, neither of which is satisfactory alone, can be combined to yield satisfactory results for certain types of alloy steels. If this belief is confirmed by the experimental work now being initiated at the Bureau, the applicability of the vacuum fusion method to alloy steels can be determined.

AMERICAN CONGRESS OF PHYSICAL THERAPY HONORS BUREAU SCIENTIST.

The American Congress of Physical Therapy has announced the award of its gold key, "the highest honor within the power of the Congress to bestow," to Dr. W. W. Coblentz, chief of the radiometry section of the National Bureau of Standards, and member of the Council on Physical Therapy of the American Medical Association "for meritorious service to medical science in the field of ultraviolet radiation."

A Street Car De Luxe.—(Science Service.) Although Prof. C. F. Hirschfeld of the Detroit Edison Co. had had no experience in designing street cars, four years research on his part with an expenditure of one million dollars, has resulted in a street car riders' "dream" of the millennium. After finding out just what riders did not like about the present type of street car and consulting with the electrical companies and car manufacturers, the designer has evolved a vehicle containing many of the conveniences already to be found in other units of transportation. Forced ventilation is supplied, the seats all face forward and are of the familiar motor bus type; deep, soft and upholstered at side and back in leather. The windows move easily, being operated like those in automobiles. The first high step getting into the car has been replaced by stairs like those of the home. Wherever a passenger stands in the car, there is a handrail or grip so near that not even one step is necessary to reach it. Lighting is indirect and provides three times normal street car illumination, without glare.

Noise is diminished within the car by replacing steel coiled springs with rubber pads. Since rubber tires on the car wheels would not stand the strain of service, the development engineers have produced a resilient wheel made of steel parts interlocked with rubber.

Many a motorist's face may be red if he tries to pass this street car from a standing start. This latest creation will accelerate about twice as rapidly as has been customary, yet it can do this far more smoothly than do present street cars. With faster acceleration in the new car, 10 per cent. more than that possible in automobiles, it is believed motor cars will be unable to cross over to the tracks because the street car will already be there.

C.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, OCTOBER 17, 1934.

The regular monthly meeting of The Franklin Institute was called to order at eight fifteen o'clock P.M. by the President, Mr. Nathan Hayward, who immediately called upon the Secretary for the business of the evening.

The Secretary reported that the full proceedings of the last regular meeting of the Institute, the Medal Day Meeting, had been printed in full in the JOURNAL OF THE INSTITUTE for July and August and expressed the wish that unless some objection were offered the President would declare the minutes approved as printed. The President stated that no objection was offered and he therefore declared the minutes approved.

The Secretary then made a report of the increase of membership during the last few months and pointed out a very significant increase in the number of Student Members in the Institute. He spoke further about attendance at the museum and the planetarium and especially concerning the very slight attendance by members of the Institute and urged them to attend more frequently.

There being no further business the President introduced the speaker of the evening, Mr. Guy Marriner, pianist and composer, who spoke delightfully on the subject "The Contributions of Science to the Development of the Modern Grand Pianoforte." Mr. Marriner's talk was accompanied by demonstrations upon the clavichord and the piano and was followed by a brilliant recital by him upon the piano. The meeting was regarded by all as a most successful one and the attendance was the largest we have yet had in the new lecture hall.

The meeting adjourned at nine-fifty-five P.M.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, October 10, 1934.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 10, 1934.

MR. CHARLES D. GALLOWAY *in the Chair.*

The following reports were presented for first reading:

No. 2954: High Power Microscopy.
No. 2984: Warren Self-Starting Synchronous (Telechron) Motors.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

RESIDENT LIFE MEMBERSHIP.

MRS. CHARLES FEARON, 6720 Emlen Street, Philadelphia, Pa.

RESIDENT.

MR. PERCY C. BELFIELD, Instructor of Mechanic Arts, Overbrook High School, Overbrook, Pa. For mailing: Box 273, Swarthmore, Pa.

DR. WALTER E. LAWSON, Director, Eastern Laboratory, Drawer B, Gibbstown, N. J.

MUSEUM MEMBERSHIP, INDIVIDUAL.

MR. RUSSELL DUANE, 1617 Land Title Building, Philadelphia, Pa.

STUDENT.

MR. DAVID H. BISHER, Bethayres, Pa.
MR. SAMUEL BOSNIAK, Philadelphia, Pa.
MR. FREDERICK W. DASCH, Philadelphia, Pa.
MR. C. WILLIAM DROSS, Philadelphia, Pa.
MR. SIDNEY ELFANT, Philadelphia, Pa.
MR. LLOYD R. FORCEY, Philadelphia, Pa.
MR. JOHN J. GRAMMARIA, Riverside, N. J.
MR. ROBERT HERR, Philadelphia, Pa.
MR. HARRY B. HERSH, Philadelphia, Pa.
MR. ROBERT S. JOHNSON, Philadelphia, Pa.
MR. DAVID F. KOHN, Philadelphia, Pa.
MR. EMELENE M. LACY, Philadelphia, Pa.
MR. FRANCIS W. LANARD, Philadelphia, Pa.
MR. ROBERT E. LENTZ, Philadelphia, Pa.
MR. JAMES E. LUCKMAN, Philadelphia, Pa.
MR. HERMAN NEWSTEIN, Philadelphia, Pa.
MR. CHARLES F. SACKETT, Philadelphia, Pa.
MR. JOHN SCHAEFFER, Philadelphia, Pa.
MR. SAMUEL SILVER, Philadelphia, Pa.
MR. PETER B. TRUMP, Merion, Pa.
MR. BENIAH L. WHITMAN, Philadelphia, Pa.

CHANGES OF ADDRESS.

LIEUT. COM. F. W. BELTZ, 3227 Thirty-third Place, N.W., Washington, D. C.
MR. W. G. H. FINCH, Woodley Park Towers, 2737 Devonshire Place, N. W. Washington, D. C.
MR. JOHN GARDINER, JR., 127 Edward Street, Philadelphia, Pa.
MR. J. H. GRANBERY, 15 East Tenth Street, New York City, N. Y.
MR. JOSEPH H. HECHT, 236 Strathmore Road, Brookline, Pa.
DR. L. M. HENDERSON, 609 Sheridan Road, Evanston, Illinois.
MR. R. K. HONAMAN, Bell Telephone Laboratories Inc., 463 West Street, New York City, N. Y.
MR. ARTHUR W. LOWE, 9304 Bustleton Avenue, Bustleton, Philadelphia, Pa.

MR. FRANK S. MACGREGOR, 2307 Ridgwayroad, Wilmington, Delaware.
MR. FRANKLIN P. MCCONNELL, Town House, Nineteenth and Spruce Streets, Philadelphia, Pa.
MR. RANDAL MORGAN, 3D, 7920 Stenton Avenue, Chestnut Hill, Philadelphia, Pa.
MR. GEORGE F. OTTO, 1029 West Cambria Street, Philadelphia, Pa.
MR. CHARLES W. REISSE, 5321 Wayne Avenue, Philadelphia, Pa.
DR. WILLIAM WILSON, 463 West Street, New York City, N. Y.
MR. CHARLES E. ZORTMAN, 1345 Tabor Road, Philadelphia, Pa.

NECROLOGY.

Mr. G. N. Condict, Plainfield, N. J.
Mr. Alfred Crook, Lansdowne, Pa.
Mr. H. A. Metz, New York City, N. Y.
Mr. L. E. Ritter, Chicago, Illinois.

LIBRARY NOTES.

RECENT ADDITIONS.

ABRAMSON, HAROLD A. Elektrokinetic Phenomena and Their Application to Biology and Medicine. 1934. American Chemical Society Monograph no. 66.

ADAMS, ROGER, AND JOHN R. JOHNSON. Elementary Laboratory Experiments in Organic Chemistry. Revised edition. 1933.

American Library Association. A. L. A. Catalog 1926. An Annotated Basic List of 10,000 Books. Edited by Isabella M. Cooper, 1926.

American Society of Heating and Ventilating Engineers. Transactions 1931–1932, Volumes 37–38. 1934.

BATEMAN, JOHN H. Highway Engineering: a Textbook for Students of Civil Engineering. Second edition. 1934.

BODANSKY, MEYER. Introduction to Physiological Chemistry. Third edition, rewritten and reset. 1934.

BOHR, NIELS. Atomic Theory and the Description of Nature. Four Essays with an Introductory Survey. 1934.

BOWDITCH, NATHANIEL. New American Practical Navigator: Being an Epitome of Navigation; First Stereotype (fourth) edition. 1817.

BROWN, H. P., AND A. J. PANSHIN. Identification of the Commercial Timbers of the United States. First edition. 1934.

BROWN, VICTOR J., AND CARLETON N. CONNER. Low Cost Roads and Bridges. 1933.

BRÜCHE, E., UND O. SCHERZER. Geometrische Elektronenoptik. Grundlagen und Anwendungen. 1934.

BURRARD, GERALD. Identification of Firearms and Forensic Ballistics. 1934.

Chemisch-technische Entwicklung auf dem Gebiete der Kohlenwasserstofföle, 1928–1932. Herausgegeben von Maximilian Pflücke. 1934.

COTTON, H. Design of Electrical Machinery. 1934.

DEN HARTOG, J. P. Mechanical Vibrations. First edition. 1934.

DEWEY, MELVIL. Decimal Classification and Relative Index. Edition 13. 1932.

EATON, LUCIEN. Practical Mine Development and Equipment. First edition. 1934.

Electrochemical Society. Transactions. Volume LXV. 1934.

FAIRES, VIRGIL MORING. Design of Machine Elements. 1934.

FISCHL, VIKTOR, UND HANS SCHLOSSBERGER. Handbuch der Chemotherapie. Erster Teil; Metallfreie organische Verbindungen; Zweiter Teil: Metallderivate. 1932–1934.

GRIFFITHS, ROOSEVELT. Thermostats and Temperature Regulating Instruments. 1934.

HENNEY, KEITH. Electron Tubes in Industry. First edition. 1934.

INGLIS, C. E. A Mathematical Treatise on Vibrations in Railway Bridges. 1934.

KARAPETOFF, V. Experimental Electrical Engineering and Manual for Electrical Testing for Engineers and for Students in Engineering Laboratories. Vol. 1. Revised by Boyd C. Dennison. Fourth edition, completely revised and reset. 1933.

KRAUCH, C. Naturforscher: grosse Physiker, Chemiker und Biologen. 1933.

LAING, J., AND R. T. ROLFE. A Manual of Foundry Practice. 1934.

LAMB, JOHN. The Operation of Motorship Auxiliary Machinery. Volume I. 1934.

LUCKIESH, MATTHEW. Seeing and Human Welfare. 1934.

MASTERS, DAVID. When Ships Go Down: More Wonders of Salvage. 1934.

MAURY, M. F. A New Theoretical and Practical Treatise on Navigation. 1836.

MOSELEY, SYDNEY A., AND H. J. BARTON CHAPPLE. Television To-day and To-morrow. Fourth edition. 1934.

RANDALL, J. T. The Diffration of X-Rays and Electrons by Amorphous Solids, Liquids and Gases. 1934.

REEDY, J. H. Elementary Qualitative Analysis for College Students. Second edition. 1932.

SCHMIDT, ALBRECHT. Die industrielle Chemie in ihrer Bedeutung im Weltbild und Erinnerungen an ihren Aufbau. 1934.

SPIELMANN, PERCY EDWIN, AND ERNEST JOHN ELFORD. Road Making and Administration. 1934. Road Makers' Library, Volume 1.

STATHER, FRITZ. Haut-und Lederfehler. 1934.

STETSON, HARLAN TRUE. Earth, Radio and the Stars. 1934.

STONER, EDMUND C. Magnetism. No date.

TOLMAN, RICHARD C. Relativity, Thermodynamics and Cosmology. 1934.

TSCHIRCH, A., AND ERICH STOCK. Die Harze. Die botanischen und chemischen Grundlagen unserer Kentnissen über die Bildung, die Entwicklung und die Zusammensetzung der pflanzlichen Exkrete. Dritte umgearbeitete Auflage von A. Tirsch. Die Harze und die Harzbehälter. Band I. 1933.

UREN, LESTER CHARLES. Petroleum Production Engineering. Oil Field Development. Second edition. 1934.

URQUHART, LEONARD CHURCH. Civil Engineering Handbook. First edition. 1934.

WIEN, W., AND F. HARMS. Handbuch der Experimentalphysik. Bd. 17, 1 Teil: Schwingungs-und Wellenlehre. Ultraschallwellen. 1934.

WILSON, W. The Calculation and Design of Electrical Apparatus. 1934.

World Power Conference. Sectional Meeting. Scandinavia. Transactions. 1933. Volume 1. 1934.

ZECHMEISTER, L. Carotinoide: ein biochemischer Bericht über pflanzliche und tierische Polyenfarbstoffe. 1934.

ZIMMER, FRITZ. Nitrocellulose Ester Lacquers: their Composition, Application and Uses. Translated by H. K. Cameron. 1934.

BOOK REVIEWS.

BEFORE THE DAWN, by John Taine, 247 pages, 14.5 × 21 cms. Baltimore, The Williams & Wilkins Company, 1934. Price $2.00.

Here is a yarn that will delight the readers of fiction with a flare for the scientific. There is a generation in the making which spends hours stretched out flat on the floor gazing at the adventures of Buck Rogers and his devastating ray. These readers will devour "Before the Dawn" as it fits into the beginnings of the pattern.

John Taine, as Eric Temple Bell, Professor of Mathematics at the California Institute of Technology, appears to have satisfied a literary appetite by the recoil effect. Wearied with the exactness of arithmetical equations, he has boiled into paleontology with a stir that should disturb every unresurrected skeleton of every prehistoric monster. One can cry bravo for the past-president of the Mathematical Society of America and member of the National Academy of Sciences. A better jubilee has not been staged since Sherman marched his troops from Atlanta to the sea. It is not with bugle and drum, however, that the ancient world of claw and scale is revived but by the ingenious device of the televisor. This instrument makes possible the television of time so that a fossil placed in position will reveal in a three dimension image all that happened during the period of his or her fossil-making time. The skeleton of a saurian can raise horse feathers on an egg shell, just as in life, and so on ad infinitum.

If television were not an actual fact, "Before the Dawn" would be funny. I have under my hand the newspaper account of Philo T. Farnsworth's television device demonstrated recently to Nathan Hayward, president of The Franklin Institute, Mayor Moore and others. The picture shows Farnsworth demonstrating his instrument for transmitting images in tiny segments at the rate of one-thirtieth to one-sixtieth to a second. With this evidence at hand one need not sniff too hard at the televisor of Taine's imagination.

It is a little odd that an authority in mathematics should suffer such a very bad attack of "adjectiveitis." After the introduction, when the engineers are perfecting their apparatus, the animals begin to arrive. The first entrant is a "beautiful little creature with her tawny, hairless skin mimicking the desert sands, and with her quick, enquiring brown eyes." At first I inclined to stumble over a skin that mimicked desert sand and eyes that come to us "quick, enquiring and brown." On the next page I found this little beauty with her "long, whip-lash tail" resting behind her on the sand in a "graceful sinusoid." And so the reptiles

are introduced to the reader out of "snaky eggs." One little devil in particular runs through the book on three terrible toes on each foot. He is dubbed Belshazzar.

And so vast periods of time are treated under the skill of engineers in order to give the entertained a sense of the seas, the shifting of land masses, the periodic upheavals, the lash of erupting volcanoes and the death of continents. The scenes are vitalized by the endless fights of the huge reptiles. That is not quite fair as the "vast, flat, vacuous faces" of the water animals are materialized as they feed on lush cress and suck in tons of unnourishing vegetation into their invisible interiors. (Maybe an interior should be identified as invisible as we should develop digestive apparatus of cellophane with the help of these same engineers.) Nevertheless, this particular feeder ended in a fight by leaping into the air and descending upon the several assailants. I presume that there is a cast of bones in some museum that supports this scene. If not, I doff my hat to Jules Verne, alias John Taine.

The most serious part of the tale is the general picture it draws of the dying continent under the effect of the approaching glaciation of the pleistocene with the drift of animal life to the south and finally the huddled route over the narrow land bridge which led to a land of warmth, safety and food. How much the saurians harassed the fugitives and under what circumstances they finally perished is a record that is in the process of composition. Reports from Florida and other termini will throw much light on the story. Maybe enough of it is available to justify the dramatic hurly-burly of the last chapters of "Before the Dawn" when Jezebel, Bartholomew, Satan and our old, young friend, Belshazzar, join in the last mix-up, with plenty of hot mud, frenzied beasts, and basaltic fire tossed in. A tender touch is found on page 208, where reference is made to the American Museum of Biological Science. The rest is scales, dagger claws, distended bellies and fights.

Just the book to give to the boy when he first scents literary gore. Not so bad for the girl who is eager to acquire a sense of biological paleontology together with geological geography. But best of all for those tired parents who in slumber will rest the head on a pillow full of slashing reptiles. But why close on a restless note, after all there were the "ratlike, hairless mammals" which managed to survive. Why spoil the dream?

"From the smoking cones our eyes dropped to the azure expanse at our feet. This beautiful water had variety and character. Never had we seen such marvelous blues as those of the lazy whirlpools which dredged up the brilliant sands from the floor of the lake and spun them into lazy patterns over the surface. Nor had we seen greener and fresher islands than the gems which flashed and sparkled less than a mile from the shore, as the wind set all the fronds of their glossy tree ferns tossing."

True enough—never had we.

H. W. ELKINTON.

RICHARD TREVITHICK, THE ENGINEER AND THE MAN, by H. W. Dickinson and Arthur Titley, 290 pages, plates, 16 × 25 cms. Cambridge, The University Press, 1934. Price $5.00.

As part of the centenary commemoration of Richard Trevithick, the committee charged with the affair determined upon a memorial volume bringing

before those in engineering and especially those studying the development of steam, the work of a pioneer in this field. The pantings of the iron horse are growing fainter and fainter each year, but there still lingers a certain sense of romance that clings like a wraith to the locomotive. There is something about the monster of our youth that echoes the early boyhood of Richard Trevithick.

Naturally a memorial book must of necessity be largely biographical. Plates enliven the text so that the reader is carried, almost before knowing it, into the great adventure of Trevithick's life—the application of high pressure steam to all sorts of industrial workings. Locomotion was the first road followed, then invention of the Cornish engine, sugar mills for the West Indies, the recoil engine and the screw propeller. One of the chapters describes high adventure in South America, where adventures in steam lead to the silver mines of Peru, copper prospecting in Chile with wanderings in Costa Rica and finally home to England. Meetings with Simon Bolivar and Robert Stephenson when in the Americas are of the unusual episodes in this engineer's life.

It is impossible to give even a hint of the character and work of Richard Trevithick compressed within the termini of 1771 to 1833. The advent of steam had arrived and with it had come a great change in the affairs of men. Inventions strewed the path of this life which bore upon and in several ways directed the course of the great revolution in power. The revolutionary events of the world during the span of Trevithick's life with their social and military fasces are beyond the compass of this book. It is enough to remark that the screw propeller still drives ships and the recoil gun carriage has not been eliminated.

The book is beautifully constructed, well bound, and very well illustrated with plates of machinery, drawings of all sorts of things, excellent reproductions of the man himself. The bibliography, index, appendices and family chart are available to the student. One might be reconciled to death in poverty to enjoy in retrospect such a memorial obituary. The tablet erected in Dartford Parish Church in 1902 tells much of the story. Between the side face of the inventor, above, and a relief of the engine he conceived, below, run these words:

To the glory of God and in memory of

RICHARD TREVITHICK

One of the pioneers of the great mechanical developments of the XIX century, and amongst the first inventors of the locomotive engine, of screw and paddle wheel propulsion for steamships, of the agricultural engine and many other appliances whereby the forces of nature have been utilized in the service of mankind. He died in poverty and was carried to his grave in the churchyard of S. Edmund, king and martyr, by the mechanics of Hall's Engineering Works, where he was then employed. This tablet was erected to perpetuate the memory of one whose splendid gifts shed lustre on this town, although he was not permitted to enjoy the fruits of his labour here.

H. W. ELKINTON

EXPERIMENTAL ELECTRICAL ENGINEERING AND MANUAL FOR ELECTRICAL TESTING FOR ENGINEERS AND FOR STUDENTS IN ENGINEERING LABORATORIES. Vol. I, by V. Karapetoff, Professor of Electrical Engineering, Cornell Univeristy, Revised by Boyd C. Dennison, Professor of Electrical Engineering, Carnegie Institute of Technology, 781 pages, tables, illustrations, 15 × 23.5 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd., 1933. Price $6.00.

The fourth edition of this work, coming sixteen years after the first, will disclose to the discerning reader a number of justified omissions and many improvements represented by new material and much valuable supplementary data to that already there. Practically the whole revision has been carried out by the junior author who was guided to a considerable extent by a series of questionnaires submitted to more than one hundred electrical engineering teachers throughout the country. Chapters on telephone practice and primary cells have been omitted, principally because instructors no longer lay stress upon these subjects. On the other hand, many new cuts have been made, a large number of which are "curves to scale," taken on actual equipment. Diagrams have been modernized and cuts of commercial apparatus brought up to date. Methods of vector representation are stressed in response to their constantly increasing use by engineering schools.

As might be expected, Volume One is occupied principally with the basic elements of electrical engineering. Thus, chapter one treats of the various methods for measuring resistance. This is followed by chapters on the construction of ammeters and voltmeters; their calibration; wattmeters and power-factor meters; and watthour meters, respectively. Prior to a consideration of the magnetic circuit a discussion on the reactance and resistance in A–C circuits is interposed. Two special phases of magnetism are dealt with by individual chapters. These are: permeability and hysteresis loop, and the measurement of core loss. Both the direct-current generator and direct-current motor are described in considerable detail. Their various operational characteristics are covered in chapters on: losses in direct-current machinery; opposition runs, and armature reaction and commutation. A comprehensive treatise on the transformer (and instrument transformers) is a prerequisite to subsequent chapters on the alternator which is discussed in the light of its operating features, voltage regulation and operation in parallel. Both polyphase and single-phase induction motors are included in this volume. It ends with a comprehensive survey of storage batteries, and D-C starters and controllers.

As the title implies, the book is not merely an exposition on electrical engineering but more intently a manual of instruction on the practical testing of electrical instruments, machinery and equipment. Within the 770 pages are to be found directions for conducting 168 individual tests. Experimental Electrical Engineering represents a text, manual and reference book combined in one.

T. K. CLEVELAND.

LOUD SPEAKERS, THEORY, PERFORMANCE, TESTING AND DESIGN, by N. W. McLachlan, D.Sc. (Engineering), London. 399 pages, illustrations, tables, 16 × 23.5 cms. Oxford, Clarendon Press, 1934.

Loud Speakers, instruments whose duty it is to reproduce as perfectly as possible sounds that previously have been emitted in the same room, the next

room; in a distant city, a far-away country; at the ends of the earth and eventually perhaps beyond the confines of terrestrial space. To the uninitiated, a loud speaker will appear to be a relatively simple affair believed to have sprung forth, full-fledged, from the "forehead" of necessity at expediency's behest. To the radio engineer and others engaged in the problem of sound reproduction, a loud speaker represents the culmination of many physical and mathematical studies on the mechanics of sound waves, the characteristics of vibratory sources and the properties of resonators. Back of the instrument may be found papers, reports, treatises, experimental models and performance graphs, all of which are prerequisites to the development of any correctly constructed loud speaker.

According to the author, the purpose of this book is to provide a fairly complete treatment of the theory and practice of loud speakers. In order to fit better the varied requirements of the readers, the theory and practice have been dealt with separately in so far as was practical. The theory, coming first, will appeal to the mathematical reader, who will not find it too elementary. For the readers occupied with testing and design, the latter portion of the book will prove to be especially helpful. The author has paid particular attention to the method of presenting his subject matter. From the material available he has endeavored to evolve the theory in logical sequence, and to give its practical applications. The treatment is as rigorous as possible, but of such a nature as will appeal to engineers.

In the theoretical section the material has been arranged under such captions as: Principles of Sound Propagation, Accession to Inertia, Vibrational Modes, Spatial Distribution of Sound from Vibrating Diaphragms, Acoustic Power Radiated from Vibrating Surfaces, Theory of Moving-Coil Principle, Hornless Speaker Simulated by Coil-Driven Rigid Disk, Electrostatic Speakers, Theory of Horns, Sound Waves of Finite Amplitude, and Transients. The practical part is presented under headings such as: Driving Mechanisms, Magnets, Electrical Impedance Measurements, Response Curves, Measurement of Vibrational Frequencies of Conical Shells, Design Considerations in Hornless Moving-Coil Speakers, and Design of Horn Type Moving-Coil Speakers.

The volume is well supplied with line diagrams and graphs. In a work of this type, such illustrations will greatly assist the reader. There is also a comprehensive list of references to aid those interested in pursuing the subject further.

T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 483, Effect of Moderate Air Flow on the Distribution of Fuel Sprays After Injection Cut-Off, by A. M. Rothrock and R. C. Spencer, 23 pages, plates, 20 × 26 cms. Washington, Superintendent of Documents, 1934. Price ten cents.

High-speed motion pictures were taken of fuel sprays with the N. A. C. A. spray-photographic apparatus to study the distribution of the liquid fuel from the instant of injection cut-off until about 0.05 second later. The fuel was injected into a glass-walled chamber in which the air density was varied from 1 to 13 times atmospheric air density (0.0765 to 0.99 pound per cubic foot) and in which the air was at room temperature. The air in the chamber was set in motion by means of a fan, and was directed counter to the spray at velocities up to 27 feet per second.

The injection pressure was varied from 2,000 to 6,000 pounds per square inch. A 0.020-inch single-orifice nozzle, an 0.008-inch single-orifice nozzle, a multiorifice nozzle, and an impinging-jets nozzle were used. The results show that in still air the dispersion of the fuel particles following injection cut-off was extremely slow and that the fuel tended to travel across the chamber from the injection nozzle. At all the air densities used, air velocities as low as 15 to 20 feet per second had an appreciable effect on the distribution of the liquid fuel after injection cut-off. The best distribution was obtained by the use of air flow and a high-dispersion nozzle.

Report No. 486, Infrared Radiation from Explosions in a Spark-Ignition Engine, by Charles F. Marvin, Jr., Frank R. Caldwell, and Sydney Steele, 14 pages, illustrations, 20 × 26 cms. Washington, Superintendent of Documents, 1934. Price ten cents.

The variations in intensity and spectral distribution of the radiant energy emitted by the flames during normal and knocking explosions in an engine have been measured at the Bureau of Standards with the financial assistance of the National Advisory Committee for Aeronautics. Radiation extending into the infrared to about 11μ was transmitted by a window of fluorite, placed either near the spark plug or over the "detonation zone" at the opposite end of the combustion chamber. Concave, surface-silvered mirrors focused the beam, first at the slit of a stroboscope which opened for about 2° of crank angle at any desired point in the engine cycle, and then upon the target of a sensitive thermocouple for measuring radiation intensity. Spectral distribution of the radiant energy was determined by placing over the window, one at a time, a series of five filters selected with a view to identifying, as far as possible without the use of a spectrograph, the characteristic emissions of water vapor, carbon dioxide, and incandescent carbon.

The results indicated that nearly all the radiant energy was in the infrared region and was emitted by water vapor and carbon dioxide formed during combustion. During normal combustion, there was evidence that reactions producing water vapor and carbon dioxide continued for 20° or more of engine crank rotation after inflammation of a given element of charge, whereas the reaction period was much shorter in the portion of the charge involved in fuel knock. The full significance of the data is at present obscured by lack of fundamental information regarding the effects of flame depth, density, temperature, and pressure, much of which could be secured by suitable observations of burner and engine flames using as fuels hydrogen and carbon monoxide separately and in known mixtures.

Report No. 488, Heat Transfer from Finned Metal Cylinders in an Air Stream, by Arnold E. Biermann and Benjamin Pinkel, 22 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1934. Price ten cents.

This report presents the results of tests made by the National Advisory Committee for Aeronautics to supply design information for the construction of metal fins for the cooling of heated cylindrical surfaces by an air stream.

Heat-transfer coefficients were obtained over a range of air speeds from 30 to 150 miles per hour from tests in a wind tunnel of a series of electrically heated

finned steel cylinders, which covered a range of fin pitches from 0.10 to 0.60 inch, average fin thicknesses from 0.04 to 0.27 inch and fin widths from 0.37 to 1.47 inches. Tests were also conducted on a smooth steel cylinder without fins.

The quantity of heat dissipated, calculated from a theoretical equation using the surface heat-transfer coefficients found from the experiments, was compared with the experimentally determined quantity of heat dissipated. The agreement was found sufficiently good to justify the use of the theoretical formula for calculating the quantity of heat dissipated from the finned metal cylinders.

A method is presented for determining fin dimensions for a maximum heat transfer with the expenditure of a given amount of material for a variety of conditions of air flow and metals.

Report No. 490, The Weathering of Aluminum Alloy Sheet Materials Used in Aircraft, by Willard Mutchler, 35 pages, 20 × 26 cms. Washington, Superintendent of Documents, 1934. Price fifteen cents.

The investigation of the corrosion of aluminum alloy sheet materials used in aircraft was begun at the Bureau of Standards in 1925, at the request of its sponsoring governmental agencies, viz., the Bureau of Aeronautics, Navy Department, the Army Air Corps, and the National Advisory Committee for Aeronautics. It has for its purpose the study of the causes of corrosion embrittlement in duraumin-type alloys and the development of methods for its elimination.

The present report contains results, obtained in an extensive series of weather-exposure tests, which reveal the extent to which the resistance of the materials to corrosion was affected by variable factors in their heat treatment and by the application of various surface protective coatings. The results indicate that the sheet materials are to be regarded as thoroughly reliable, from the standpoint of their permanence in service, provided proper precautions are taken to render them corrosion-resistant.

Report No. 497, Computation of the Two-Dimensional Flow in a Laminar Boundary Layer, by Hugh L. Dryden, 11 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1934. Price five cents.

A comparison is made of the boundary-layer flow computed by the approximate method developed by K. Pohlhausen with the exact solutions which have been published for several special cases. A modification of Pohlhausen's method has been developed which extends the range of application at the expense of some decrease in the accuracy of the approximation.

The work was carried out at the National Bureau of Standards, in part with the coöperation and financial assistance of the National Advisory Committee for Aeronautics.

Report No. 501, Relative Loading on Biplane Wings of Unequal Chords, by Walter S. Diehl, 8 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1934. Price five cents.

It is shown that the lift distribution for a biplane with unequal chords may be calculated by the method developed in N. A. C. A. Technical Report No. 458 if corrections are made for the inequality in chord lengths. The method is applied

to four cases in which the upper chord was greater than the lower and good agreement is obtained between observed and calculated lift coefficients.

ELECTROKINETIC PHENOMENA AND THEIR APPLICATION TO BIOLOGY AND MEDICINE. By Harold A. Abramson, M.D. American Chemical Society Monograph No. 66. 331 pages, with 57 tables and 106 figures. 15.5 × 23.5 cm. The Chemical Catalog Company, Inc., New York, 1934. Price $7.50.

The processes of arriving at scientific generalizations with respect to phenomena occuring at surface boundaries have taxed the ingenuities of hundreds of most resourceful investigators in chemistry and physics. It is a domain in which physical chemistry reaches its apotheosis. If one paused to consider the variety of familiar objects, each with its grosser individual characteristics, not to mention unnumbered microscopic and submicroscopic variations, then to consider that every phase boundary is the seat of energy, he might conclude that the task of introducing any law and order with respect thereto quite hopeless. Not merely are inanimate surfaces involved. It is still more important that all living tissues are influenced by their environment and energy relations. Dr. Abramson's volume is an important step towards the organization of surface phenomena through the approach offered by electrokinetic measurements. While the treatment is somewhat less than convincing in places and frankly incomplete in others he has accomplished much good in correlating an enormously diversified mass of observations.

The contents divide into an historical chapter, one on early theory and experiments, an important and probably too brief chapter on experimental methods, and another on recent developments. The remainder of the volume deals with more specialized matters and comprises chapters on proteins and related compounds, on the general effects of salts on "inert" surfaces, one each on organic and inorganic surfaces, a few pages on gases and more extended treatments relating to blood cells, tissues and the like and bacteria, antibodies, viruses and related systems.

The discovery of electroosmosis dates back to Reuss (1808), following closely the decomposition of water by means of galvanic current (1800), and the isolation of sodium and potassium by Davy (1807). Early experiments on electroosmosis, electrophoresis and streaming potentials are described interestingly and at some length.

Two chapters lay the mathematical foundation. The Helmholtz double layer theory, subsequently reinvestigated and extended by Smoluchowski, are dealt with fully, likewise the more modern diffuse double layer theory of Gouy. The Debye and Hückel approximation is derived and the correlation between it and the Gouy theory shown.

Chapter III deals with experimental measurements of electrokinetic potential, the most space being given to microscopic and ultramicroscopic observation methods. The deflection method of Sumner and Henry, the moving boundary method, the transference method, streaming potentials, sedimentation potentials and electroosmosis measurements are considered briefly.

The practical aspects of electrokinetic phenomena are brought to the front in the study of proteins and organic surfaces in general. The work of Gortner and associates on cellulose has been given prominence. Enormous fields for

further experimentation with respect to inorganic and organic surfaces as well as living tissues still exist. The important matter of base exchange is not mentioned in the index and only incidentally in the text although it is a surface phenomenon of rather wide spread occurrence.

A great number of subjects are touched on in connection with living matter but usually it will be necessary to consult the original references to get a comprehensive view of the work done. This statement applies almost equally to the sections on organic and inorganic surfaces and to the sections on bacteria, viruses and the like. The references are strictly up-to-date.

Commercial applications of electrokinetic phenomena have had to await the development of science itself. Rapid progress has been made since the modern theories of solution and surface structure have been put forth. Important industrial applications may follow rapidly. Appendix III lists patents dealing with a number of technical applications such as the drying of peat, electro-deposition of latex, separation of water and oil emulsions, impregnation and tanning, and the purification of clay, water, sugar solutions, glycerine, gelatine, serum and gases.

LESLIE R. BACON.

THE MYSTERIES OF THE ATOM, by H. A. Wilson, M.A., M.Sc., F.R.S., 146 pages, illustrations, 16 × 23.5 cms. New York, D. Van Nostrand Co., Inc., 1934. Price $2.50.

Though the number of physicists in this world may be relatively small, the number of persons wishing to acquire a general understanding of modern physical thought is, on the other hand, very large. The author of this book, a member of the elect minority, has heeded the ever present demand of an intelligent layman throng for more and better essays on most recent in physical discovery and has written in a language comprehensible to all.

The title itself is arresting. Between these covers is to be found an explanation of the "Mysteries of the Atom." One is impelled to at least glance over the Table of Contents to see what may be in store for the earnest reader. The book begins with a recitation of more familiar things. The status of physical science in the nineteenth century is described. This provides the basis upon which is set the gradual evoluted structure of modern physical thought. Possibly the electron and proton and the part they play in atomic structure are familiar to many readers. Nevertheless they are part of the picture and will continue to assist future attempts to reconcile waves and particles, light and photons.

The "Uncertainty Principle" has become a byword among physicists and a "New Theory of Atoms" is the necessary outgrowth. Cosmic Rays can no longer be ignored even though their origin and constitution are still food for polemical thought. Space, Time, Relativity and Gravitation are no longer the concrete concepts we once were led to believe. They are almost as imaginary as the square root of minus one. For the reader who has had some training in the mathematical and physical sciences, a group of appendices is included at the end of the book. These are chiefly descriptions of various critical experiments which represent "mile-stones" in the advance of modern physics.

T. K. CLEVELAND.

SOCIETY OF DYERS AND COLOURISTS, Report on the Work of Its Fastness Committee in Fixing Standards for Light, Perspiration and Washing, 51 pages, illustrations, tables, 14 × 22 cms. Yorkshire, Society, 1934.

A brochure which gives little indication of the immense amount of work and effort that has been expended in its production. The Society of Dyers and Colourists Committee on Fastness has performed quite an accomplishment in developing a set of standards whereby dyers may determine more accurately the resistance of dyed goods to light, perspiration and washing.

A set of red and blue standards, each set composed of seven pieces of dyed cloth, representing seven degrees of fastness, have been developed and will be used for determining by comparison the resistance of other dyes to fading. In connection with establishment of the color standards, the Committee has studied carefully various sources of artificial illumination in order to determine which shall best reproduce natural fading conditions.

Possibly the development of the method for determining fastness to perspiration presented the greatest number of difficulties. First, the composition of sweat had to be established, then a synthetic preparation found which should react toward colors in an identical manner. The testing routine finally recommended is somewhat lengthy but is justified on the basis of its reliability.

The tests for determining fastness to washing are very comprehensive in scope and aim to duplicate both household and commercial laundering conditions. Whereas the standards for light and perspiration consisted of dyed worsteds, those for the washing tests are more appropriately prepared from unmercerized cotton and number five to a set, both red and blue as before.

In the Appendix will be found a more detailed description of the mechanical devices recommended for comparative testing. The Fastness Committee will supply the textile dyer with approved sets of the standards so that he may classify his goods uniformly with those of other manufacturers. For single tests, the Wool Industries Research Association will conduct such, thus saving many the unwarranted expense of installing the necessary apparatus.

T. K. CLEVELAND.

JÖNS JACOB BERZELIUS, AUTOBIOGRAPHICAL NOTES, published by the Royal Swedish Academy of Sciences through H. G. Soderbaum, translated from the Swedish by Olof Larsell. 194 pages, plates, 14 × 21 cms. Baltimore, Williams and Wilkins Company, 1934. Price $2.50.

The field of chemistry and the realm of science in general owe a debt of gratitude to the Swedish Academy of Sciences. For did not this learned society decree in 1820 that each member should present a biographical sketch and was not Jöns Jacob Berzelius a member who conscientiously fulfilled his assignment? Not only that, but about twenty years later, in 1842, this famed chemist revised and brought up to date his remarkably human autobiographical document. This book is nothing more or less than a very careful translation of Berzelius' writings from the original Swedish by Professor Olof Larsell.

The story, as told, will sustain the reader's interest to a degree greater than the mere title might lead one to expect. For the great Berzelius in his introduction, remarks: "That which I here give is in truth more complete than the Acad-

emy really requires. . . . The numerous biographical sketches concerning me, published in foreign book, which I have had occasion to read during my lifetime, however flattering their content may be for me, have nevertheless contained a multitude of altogether incorrect statements which require correction. . . ."

If Berzelius had been no more than a chemist, the appeal in his story probably would have suffered. He was, however, physician, business man, teacher and politician, but on a scale less grand. It is not easy to believe that a biographer ever could have written anything which gives a better insight into the character of Berzelius than the man himself did. He straightforwardly tells of his early struggles for economic independence, and the obstacles encountered in his desire to study chemistry. His unbiased description of his contemporaries promotes nothing but admiration. A description of his own accomplishments is modesty itself.

The translator has omitted what might seem to be irrelevant material in an English version. These deleted sections consist of notes on the Academy of Sciences and the list of royal orders and learned societies of which Berzelius was a member. Mention must be made of the many interesting notes and parallel passages which have been included by the editor of the original Swedish edition. Their context is as interesting as the book proper and adds much to acquiring a proper appreciation of a great character.

T. K. CLEVELAND.

A TEXTBOOK OF ORGANIC CHEMISTRY, by Joseph Scudder Chamberlain, Ph.D., Professor of Organic Chemistry, Massachusetts State College, third edition revised. 873 pages, 15 × 22 cms. Philadelphia, P. Blakiston's Son & Co., Inc., 1934. Price $4.00.

In this edition the text has been divided into three parts. Part I covers 366 pages and is devoted to the aliphatic series. Part II covers 260 pages and includes both the carbocyclic and the heterocyclic series. Part III, entitled "Supplementary Topics," is a new feature of the treatise; it contains 174 pages of material drawn from all three series of organic compounds, and is intended to supply the class with additional material for study when possible or desirable. The topics in this part are: Petroleum, Industrial Carbohydrates (including sugar and cellulose), the Amino Acids and Proteins, Coal Tar, Reactions of the Diazo Compounds (including a tabular resumé), Dyes, Terpenes and Alkaloids (including ptomaines).

The value of the book is increased by a bibliography (2 pages) of treatises on organic chemistry. One appendix contains a concise 15-page summary of the procedure for the separation, purification, identification, analysis, and determination of the molecular weight of organic compounds. Another appendix gives bibliographical references to procedures for the laboratory preparation of 426 organic compounds and to the laboratory procedures in certain analytical determinations.

In the text, the suffix of Geneva names ending in *ine* has been changed to *yne*, thus ethine or acetylene is designated eth*yne*.

The treatise is comprehensive, and will be of service to the physician and biologist, as well as to the chemist.

JOSEPH S. HEPBURN.

RECENT ADVANCES IN ATOMIC PHYSICS, by Gaetano Castelfranchi, Professor in the High School for Engineers in Milan. Approved translation by W. S. Stiles, Ph.D., and J. W. T. Walsh, M.A., D.Sc. Vol. I, Atoms, Molecules and Electrons. Vol. II, Quantum Theory, illustrations, 14 × 20.5 cms. Philadelphia, P. Blakiston's Son & Co., Inc., 1932. Price $4.00 each.

Although an opportunity to go over these two interesting volumes has presented itself only but recently, like many good things, nothing has been lost through their aging. In these times, one volume offers hardly sufficient space to a writer who attempts to cover the subject of Physics with any appreciable degree of adequateness. Faced with this condition, the author has congregated the material manifestations, such as atoms, molecules and electrons into Volume One and reserved the Quantum Theory abstractions for Volume Two.

In preparing this work, the author seems to have had in mind the needs of students planning to specialize in Physics or just as appropriately, graduate students majoring in other sciences and who have recourse to Physics as a so-called minor subject. The character of the work is aptly expressed by its author: " . . . rigidly adhered to the simplest possible form of treatment, avoiding all that is abstruse, all unnecessary elaborations of the argument, and all lengthy calculations which hinder a rapid comprehension of the subject as by obscuring it in a mass of detail." In other words, the reader who is acquainted with the general principles of physics and mathematical sciences, will find that the books need no expositor.

The topics of "Atomic Physics" are many and interrelated and at times the author must have wondered just where to insert a chapter heading. However, the subject matter follows an orderly arrangement and Volume One starts out with a more or less introductory discussion of Atoms and Molecules in Physical Chemistry terminating with a chapter on the heart of matter—the Atomic Nucleus. Light and the Kinetic Theory of Gases are subjects of basic importance in a work of this type. A brief chapter on Fluctuations covers a number of infrequently discussed phenomena. Electrons and Positive Rays are the Precursors of Isotopes, while X-rays have begotten Atomic Numbers and disclosed Crystal Structure. Radioactivity is a large subject and its condensation to a brief chapter requires skill.

It is not difficult to guess the starting point of Volume Two: Thermal Radiation and Quantum Theory. Spectroscopy, Bohr's Theory and the Energy Levels of the Atoms form one subject grouping, while another includes the Stark and Zeeman Effects, Multiplet Lines and the Spinning Electron. Specific Heats warrant consideration in the light of the Quantum Theory as does the Photoelectric Effect. The Compton Effect, closely associated with Light Quants, occupies a section just preceding that on Magnetism. Philosophical trends are indicated in the chapter on wave and quantum mechanics, and the far-reaching significance of the New Statistics and their applications gives us some idea of what the future holds in store.

T. K. CLEVELAND.

HEAT, by James M. Cork, Ph.D., Associate Professor of Physics, University of Michigan. 279 pages, illustrations, tables, 15 × 24 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd., 1933.

Since "heat" is a quality of universal scope, it is not surprising that books are written upon such a subject. As is the case with most of Nature's intangibles,

science has been very deliberate, if not plain slow, in solving the mystery of heat's coming and going. True, a thousand years are but a day to the seekers of knowledge and it seems only yesterday that the phlogistonites held forth. Between the demise of "phlogiston" and the latent refinements of the Quantum Theory are volumes of data compiled by numerous investigators on the properties and nature of heat. Although the author, in "Heat," has placed emphasis upon the modern heat theories, which is quite proper, a certain amount of historical background must be included. As a result, "Heat" supplies the reader with an excellent perspective of the complete field.

In preparing "Heat," the author has evidently sought to provide a textbook which may supplement a course of instruction upon this particular phase of physics. Measurement of temperature receives first consideration. The methods and means are many with each best suited to some specific case. A chapter on Specific Heats introduces the equations of empirical origin to the equations of classical theory. The process of their reconciliation extends throughout the book. The matter of Thermal Expansion precedes a consideration of Heat Transfer by Conduction and Convection. The developing of the First Law of Thermodynamics and the establishment of Energy Equivalence takes the reader back to the "good old days" while the topic of "Radiation" becomes intensely modern once more. Change of State and Continuity of State in respect to solids, liquids and gases have many practical applications. Naturally, while on the subject of heat, the Second and Third Laws of Thermodynamics cannot be ignored and are discussed in connection with engine cycles and engine efficiencies. The production of low and high temperatures are considered in the light of the principles involved.

The instructive value of the book has been enhanced by placing in each chapter certain experiments dealing with fundamental thermal quantities. The equipment and technique are simple enough to permit satisfactory repetitions. Workers in heat problems will find in the Appendix, the greater part of all data essential to their operations.

T. K. CLEVELAND.

INDUSTRIAL HEAT TRANSFER, by Alfred Schack, Dr.-Ing., translated from the German by Hans Goldschmidt, Dr.-Ing., and Everett P. Partridge, Ph.D. 371 pages, tables, illustrations, 15.5 × 23.5 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Ltd. 1933. Price $5.00.

According to the author, the interests of the practical engineer have been particularly considered in the preparation of this book on Industrial Heat Transfer. Every attempt has been made to present heat transfer equations in their simplest forms. Where such simplification is impossible, charts have been furnished in order that the engineer may obtain the desired results without involved calculations. In line with the policy of producing a work of maximum practical value, Dr. Schack has substituted for the complicated equations based on the theory of similarity, simpler empirical equations of the greatest possible accuracy.

Completeness has not been sacrificed for the sake of practicality as evidenced by the inclusion of the derivations for the various equations. However, such sections are printed in small type and can be skipped by readers not especially interested in that phase of the subject. Extreme care has been taken in the translation of the book and the translators have attempted to preserve the meaning

of the original text while at the same time producing a composition more readable than a word-for-word literal translation. Guiding marks to the more important equations have been added and the metric system of the German text has been tediously transposed into that of the English Btu-foot-pound-degree Fahrenheit. The American Standards Association nomenclature has been followed as far as possible and the bibliography enlarged as well as brought up to date.

Slightly more than the first half of the volume is given over to a study of the three mechanisms for heat transfer: Conduction, Convection and Radiation. There is also a short section on Heat Transfer from Condensing Vapors. The calculations for heat exchangers include the parallel, counter and cross-flow types. The mechanism of heat transfer in industrial furnaces is very helpful and is supplemented by a discussion on determining an economic balance between heat transmission and pressure drop. Most valuable of all to the practical engineer is probably that section composed of Numerical Examples in Heat Transfer. These include the transmission of heat through various types of walls; calculations for water preheaters, recuperators and regenerators; radiation in reverbatory types of furnaces and heat absorption of high-pressure water walls. At the end, a compilation of the most important heat transfer equations is given. This is followed by lists of important physical constants having to do with the thermal conductivity of the metals, insulating and construction materials, solids, liquids and gases. Radiation constants also are supplied.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

Structural Design in Steel, by Thomas Clark Shedd, 560 pages, illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1934. Price $5.00.

Tautomerism, by John William Baker, 332 pages, illustrations, 13.5 × 22 cms. New York, D. Van Nostrand Company, Inc., 1934. Price $9.00.

Die Elektrischen Maschinen, Band III, Berechnung und Bemessung, von Dr.-Ing. M. Liwschitz, 409 pages, illustrations, 15 × 23 cms. Leipzig und Berlin, Verlag und Druck von B. G. Teubner, 1934. Price 22.50 R.M.

Elemente der elementaren mechanik, von K. H. Grossmann, 99 pages, illustrations, 15 × 21 cms. Zurich, Author, Price 3 francs.

Cours Pratique d'Electricite Industrielle, par Leon Bouthillon and M. Goudonnet, Premier Volume, Les lois fondamentales de l'electricite, 305 pages, illustrations, 12 × 18.5 cms. Paris, Gauthier-Villars, 1934. Price 25 francs.

Generation of the Universe and "Design for Living," by Percy A. Campbell, 101 pages, 14 × 20 cms. East Cleveland, The Caxton Company, 1934.

United States Bureau of Mines, Bulletin 378, Underfeed Combustion, Effect of Preheat, and Distribution of Ash in Fuel Beds, by P. Nicholls, 76 pages, tables, 15 × 23 cms. Washington, Government Printing Office, 1934. Price ten cents.

Bell Telephone Laboratories, Monographs: B–790, Viscous Resistance in Capillaries of Non-Uniform Radius, by D. A. McLean, 6 pages. B–804, The Determination of Dielectric Properties at Very High Frequencies, by J. G. Chaffee, 12 pages, illustrations. B–805, Phase Angle of Vacuum Tube Transconductance, by F. B. Llewellyn, 10 pages, illustrations. B–806, Ferromagnetic

Materials Heat Treated in Magnetic Fields, by G. A. Kelsall, 4 pages, tables. B–807, Proposal of a Method of Observing the Solar Corona Without an Eclipse, by A. M. Skellett, 4 pages. 5 pamphlets, 15 × 23 cms., New York, Bell Laboratories, 1934.

Chemical Engineering Catalog, 1934 (Nineteenth Annual) Edition, collected, condensed and standardized catalog data of equipment, machinery, laboratory supplies, etc. 779 pages, illustrations, 21.5 × 29 cms. New York, Reinhold Publishing Corporation, 1934.

National Advisory Committee for Aeronautics, Nineteenth Annual Report, 1933, including Technical Reports Nos. 441 to 474. 637 pages, illustrations, tables, 23 × 29.5 cms. Washington, Government Printing Office, 1934. Price $2.50.

National Advisory Committee for Aeronautics, Technical Notes, No. 505, The Effects of Full-Span and Partial-Span Split Flaps on the Aerodynamic Characteristics of a Tapered Wing, by Carl J. Wenzinger, 6 pages, illustrations, 20 × 26 cms. Washington, Committee, 1934. No. 506, Experimental Verification of Theodorsen's Theoretical Jet-Boundary Correction Factors, by George Van Schliestett, 26 pages, tables, figures, 20 × 26 cms. Washington, Committee, 1934.

CURRENT TOPICS.

Industrial Chemical Health Hazards.—(*Chem. & Ind.*, Vol. 53, p. 707.) The matter of protecting the worker's health has become increasingly important to his employer. Any indifference the latter may have shown toward this situation in the past has been eliminated by the passage of legislation protective to the worker. DR. J. GRANT CUNNINGHAM has come forth with a very timely paper on the more important chemical health hazards in industry and discusses each from the standpoint of pathological effects and physiological behavior including the methods of treatment based upon such knowledge.

Lead.—The individual with lead poisoning complains of headache, poor appetite, especially for breakfast, lassitude, persistent constipation and in the acute stage, abdominal colic, weakened hand grip and even wrist drop. An acute attack must be distinguished from acute appendicitis.

About one-fifth of a grain of lead per one thousand cubic feet of air is given as the dose which, over a period of months, will produce poisoning, so that the actual exposure necessary is small. In a sense lead is a cumulative poison. In the body, under suitable conditions of hydrogen-ion concentration, lead is taken from the blood, is converted into insoluble tri-lead phosphate, and stored in the bones. Relatively small changes in reaction serve to reverse the conditions, drawing lead back into circulation for excretion by the kidneys or through the bile. Too great and sudden a reversal will precipitate an acute attack of lead poisoning.

Individuals acutely ill are rapidly relieved by the intravenous injection of calcium. Subsequent treatment involves the administration of substances known to alter the hydrogen-ion concentration to favor the release of lead from the bones in sufficiently small amounts that it may be excreted fast enough to prevent a fresh attack of acute symptoms. This may be accomplished by the administration of acid or acid-producing salts, at the same time withholding calcium, or by the use of alkali alone. Such substances are phosphoric acid, ammonium chloride, sodium bicarbonate or parathyroid extract. If stored lead is not removed, the individual is liable to an acute attack should anything occur to change this balance, such as a seizure of influenza or alcoholic excess.

Hydrocarbons.—Of the distillation products of coal tar and petroleum benzene is the most important industrially. Benzene

will produce an effect in concentrations as low as one hundred parts per million of air. In high concentration asphyxia results but with lower repeated exposure the blood-forming organs are affected, a reduction in the white cell count occurs and a later similar effect upon the red cells. It is rather strange that toluene and xylene do not also attack the bone marrow, since they so closely resemble benzene. The former two are oxidized to benzoic and toluic acid respectively in the course of bodily disposal, while the benzene is transformed into phenol and dihydroxybenzenes.

Aniline is readily absorbed through the skin, producing methemoglobin in the blood, thereby diminishing its power to absorb oxygen. It is oxidized to para-amido phenol which is only slightly toxic. Paranitraniline and paratoluidine are excreted from the body practically unchanged. The latter substance is suspected to be the cause of bladder cancer in aniline workers. Dinitrophenol illustrates a condition where women for some reason are less affected by exposure than men.

Carbon tetrachloride and trichlorethylene are both fat solvents and therefore attack the nervous system. The former is poisonous in concentrations of about 5,000 parts per million for one hour while the trichloroethylene is rather more toxic. Of the higher boiling point distillation products the most important is the fraction thought to contain a cancer-forming substance. In coal and petroleum distillation the anthracene fraction boiling at about 300° C. is believed to contain this substance.

Hydrogen Sulphide.—The odor of hydrogen sulphide can be detected in concentrations of ten parts per million. In higher concentrations it produces a sense of oppression in the chest, headache and nausea, weakness with eventual paralysis of the respiratory center. If it is not fatal immediately, œdema of the lungs may develop even after four or five days, One hundred and fifty parts per million will produce symptoms in a few hours, while cyanide at about sixty parts per million produces a similar condition.

Cyanide.—Cyanides act by combining with some iron compound in the cell, ordinarily active in catalyzing cell oxidation, so that the tissue is unable to take up oxygen. When poisoning is caused by the ingestion of cyanide compounds the administration of methylene blue has been attended with some success. The use of sodium nitrite and amyl nitrite also is indicated. Where the poisoning is caused by cyanide inhalation, the efficacy of any known antidote is questionable.

Carbon Monoxide.—This gas, when breathed, readily combines with the blood hemoglobin to produce carboxy-hemoglobin and consequent lessened oxygen-carrying power of the blood. The only ef-

fective treatment for carbon monoxide poisoning appears to be the use of artificial respiration and the administration of oxygen-carbon dioxide mixtures.

Silica.—Of the mineral substances encountered, silica is the most important. A fibrosis of the lungs associated with tuberculosis in workmen exposed to mining dust has been recognized for centuries. It is now almost universally held that silica is dissolved in the tissue to produce a poison, the local response to which is the production of fibrosis. Talc dust is similar in its effects. How silica acts in the body to produce an increased susceptibility to tuberculosis and a more serious outlook when it does develop, is not understood.

C.

Infrared Rays Expose the Spanish Inquisition.—(Science Service.) The Franklin Institute Museum has received copies of infrared photographs of 300-year old passages censored by the Spanish Inquisition. These are on display in the Photographic Section.

In the early seventeenth century, one Theodore de Bry published in several languages an account of his travels in India. Officers of the Spanish Inquisition condemned certain passages and streaked them out with brushfuls of black ink. This obliterating ink was found to be transparent to infrared light so that, when illuminated, the ink of the original text became readily visible.

De Bry's offending passages turned out to be not particularly naughty after all—at least, not all of them. One, that got the blackest kind of an inking-out, merely offers the opinion that some of the processional ceremonies of Christian monks had been adopted from Eastern practices.

C.

Tree Cavity Filler.—(*Chemical Industries*, 35, 229.) A large rubber company, in conjunction with the Akron, Ohio, parks department, has perfected a plastic rubber tree cavity filler. The material readily adheres to the cavities or scars of damaged trees and prolongs their lives indefinitely. Supplementing the cement is a new tree paint that may be used to cover cuts resulting from pruning and trimming.

Soaping Roads.—(*Chemical Industries*, 35, 229.) In connection with the dust-proofing of low cost dirt and gravel roads, the Missouri State Highway Department has found that a sample of soil treated with soap in addition to oil, will resist water for over a year, whereas the same soil merely oil-treated, slaked down in less than a day's time. In some cases, such high protective effect was

obtained with the use of as little oil as 3 per cent. of the weight of the dry soil along with 0.3 per cent. (one-tenth as much) soap. In practical experiments, road oil and an aqueous soap solution were spread through separate spray bars but directed upon the same spot of the road surface, thereby forming the desired emulsion at that point.

C.

Keeping Rivets in Very Cold Storage.—(*Chemical Industries*, 35, 229.) Duralumin is an alloy, which, after healing, must age before obtaining its full strength and hardness. All duralumin rivets should be set before this ageing takes place. It is now customary, in airplane manufacture, to heat rivets to 950° F., quench them in water, and pack them in dry ice until needed—as much as two weeks later. Ageing sets in with normal temperatures.

C.

What Bone Glue Is Good For.—(*The Chemical Age*, 31, 257.) Some time ago Current Topics made mention of a competition fostered by the International Bone Glue Manufacturers' Association—"Epidos" for short. Apparently the contest brought results and now ten first prizes have been distributed to an even number of individuals who have described new uses or developed existing applications for bone glue. One prize recipient uses it in soap to harden it and improve the lathering properties. Another reports that the addition of 3 per cent. to 5 per cent. bone glue to cement increases its water resistance. Bone glue can be used in insecticidal sprays to improve adherence on the fruits or plant. Wax polishes of the emulsion type are stabilized with bone glue. Glue added to rubber before vulcanization improves its tenacity, durability and homogeneity. Bone glue dissolved in organic liquids such as phenols give resin-like products having good electrical insulating properties. A logical application is that of a binder in the making of insulating building material from cork, sawdust or wood shavings. Paints, lacquers and varnishes may be built upon a bone glue base, the glue being rendered insoluble with bichromate or formaldehyde at the time of application. When previously hydrolyzed by the action of sulfuric acid, bone glue serves as a source of nitrogen for growing yeast.

Not many readers may know it, but rubberized horsehair is used as seat padding particularly in motorcoach work. From Czecho-Slovakia comes a recipe for a coating with a bone glue base of ammonium resinate for preserving this horsehair filler. A par-

tially insolubilized bone glue makes an excellent paper size and may be substituted for rosin. The glue also may be used as an inhibitor for acid pickling baths.

C.

G. E. House of Magic in Philadelphia.—The Franklin Institute is pleased to announce that it is to have for exhibition purposes for two weeks, from November second to seventeenth, inclusive, the General Electric Company's famous House of Magic which has just been released from the World's Fair in Chicago. Those who visited the Fair will recall that this exhibit was one of the most attractive and most popular on the fair grounds. It is being removed intact to the Museum and will be in operation during the two weeks above-named.

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